Linearization of FAST v7

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Jason Jonkman, Ph.D.
Senior Engineer, NREL

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Outline

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• Model Linearization:
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  – 1st-Order Model
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Introduction & Background

- While linearization is not yet available in FAST v8, linearization across all coupled modules was a design feature of the modularization framework.

- FAST v7 only allows linearization of the structural model with quasi-steady aero-hydro-servo features.

- Applications of linearization:
  - Full-system modal analysis:
    - Frequencies
    - Damping
    - Mode shapes
    - Of a stationary or operational turbine
  - Controls design:
    - Develop state-space representation of wind turbine “plant”
    - Includes control inputs, wind disturbances, & output
  - Stability analysis
Introduction & Background (cont)

- Linear model is only valid in the local vicinity of an operating point (OP):
  - Requires that one select or compute an OP before linearization
- When rotor is spinning, the linear system is periodic:
  - Azimuth-averaging averages-out the periodic effects
  - Multi-blade coordinate (MBC) transformation expresses cumulative effect of blade dynamics in the fixed frame
Calculating Operating Points

• Operating-point choices:
  – Initial condition (user-selected) (watch out for nonstationary OP!)
  – Static equilibrium (parked or idling)
  – Periodic steady-state equilibrium (operational):
    • No trim: All controls fixed (fixed speed)
    • Trim: One control input varied to achieve desired rotor speed:
      – Nacelle yaw
      – Generator torque
      – Blade pitch

• Equilibrium solutions found through time-domain simulation:
  – Solution found within user-selected displacement & velocity tolerances
  – Solution can be sped up with optional compile-time feature to artificially increase system damping
  – Optional trim calculation is automated with a proportional feedback control law on rotor-speed error
Model Linearization

2nd-Order Model

Nonlinear EoM:

\[ M(q,u,t) \ddot{q} + f(q, \dot{q}, u, u_d, t) = 0 \]

Nonlinear Outputs:

\[ \text{OutData} = Y(q, \dot{q}, u, u_d, t) = Y_r(q, u, t) \ddot{q} + Y_i(q, \dot{q}, u, u_d, t) \]

Perturbation of system variables:

\[ t = t_{\text{op}} \quad q = q_{\text{op}} + \Delta q \quad \dot{q} = \dot{q}_{\text{op}} + \Delta \dot{q} \quad u = u_{\text{op}} + \Delta u \quad u_d = u_d_{\text{op}} + \Delta u_d \]

\[ \ddot{q} = \ddot{q}_{\text{op}} + \Delta \ddot{q} \quad Y = Y_{\text{op}} + \Delta Y \]

Linear EoM:

\[ M \Delta \ddot{q} + C \Delta \dot{q} + K \Delta q = F \Delta u + F_d \Delta u_d \]

Linear Outputs:

\[ \Delta Y = VelC \Delta \dot{q} + DspC \Delta q + D \Delta u + D_d \Delta u_d \]

\[ M = M_{\text{op}} \]

\[ C = \left[ \frac{\partial f}{\partial \dot{q}} \right]_{\text{op}} \]

\[ F = \left[ \frac{\partial M}{\partial \dot{q}} \ddot{q} + \frac{\partial f}{\partial u} \right]_{\text{op}} \]

\[ K = \left[ \frac{\partial M}{\partial q} \ddot{q} + \frac{\partial f}{\partial q} \right]_{\text{op}} \]

\[ F_d = -\left[ \frac{\partial f}{\partial u_d} \right]_{\text{op}} \]

\[ F = \left[ \frac{\partial M}{\partial \dot{q}} \ddot{q} + \frac{\partial f}{\partial u} \right]_{\text{op}} \]

\[ VelC = \frac{\partial Y}{\partial \dot{q}} \bigg|_{\text{op}} = \left[ -Y_r M^{-1} C + \frac{\partial Y_i}{\partial \dot{q}} \right]_{\text{op}} \]

\[ DspC = \frac{\partial Y}{\partial q} \bigg|_{\text{op}} = \left[ \frac{\partial Y_r}{\partial q} \ddot{q} - Y_r M^{-1} K + \frac{\partial Y_i}{\partial q} \right]_{\text{op}} \]

\[ D = \frac{\partial Y}{\partial u} \bigg|_{\text{op}} = \left[ \frac{\partial Y_r}{\partial u} \ddot{q} + Y_r M^{-1} F + \frac{\partial Y_i}{\partial u} \right]_{\text{op}} \]

\[ D_d = \frac{\partial Y}{\partial u_d} \bigg|_{\text{op}} = \left[ Y_r M^{-1} F_d + \frac{\partial Y_i}{\partial u_d} \right]_{\text{op}} \]

(matrix sizes determined by enabled DOFs)
Model Linearization

1st-Order Model

Conversion From 2nd Order to 1st Order:

\[
\begin{align*}
\mathbf{x} &= \begin{bmatrix} \Delta q \\ \Delta \dot{q} \end{bmatrix} \\
\dot{\mathbf{x}} &= \begin{bmatrix} \Delta \dot{q} \\ \Delta \ddot{q} \end{bmatrix} \\
\mathbf{y} &= \Delta \mathbf{Y}
\end{align*}
\]

Linear EoM & Outputs:

\[
\begin{align*}
\dot{\mathbf{x}} &= A \mathbf{x} + B \Delta \mathbf{u} + B_d \Delta \mathbf{u}_d \\
\mathbf{y} &= C \mathbf{x} + D \Delta \mathbf{u} + D_d \Delta \mathbf{u}_d
\end{align*}
\]

\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ M^{-1}F_d \end{bmatrix}, \quad C = [DspC \hspace{1cm} VelC]
\]
Model Linearization
Numerics

- Linearization done numerically with central-difference method:
  - Perturbations made about OP
  - Perturbations are hard-coded:
    - Default perturbation: 2°
    - Can be changed at compile time

\[-M^{-1} C = \frac{\ddot{q}\left(q_{op} + \Delta q\right) - \ddot{q}\left(q_{op} - \Delta q\right)}{2 \Delta q}\]

\[-M^{-1} K = \frac{\ddot{q}\left(q_{op} + \Delta q\right) - \ddot{q}\left(q_{op} - \Delta q\right)}{2 \Delta q}\]

\[M^{-1} F = \frac{\ddot{q}\left(u_{op} + \Delta u\right) - \ddot{q}\left(u_{op} - \Delta u\right)}{2 \Delta u}\]

\[M^{-1} F_d = \frac{\ddot{q}\left(u_d_{op} + \Delta u_d\right) - \ddot{q}\left(u_d_{op} - \Delta u_d\right)}{2 \Delta u_d}\]

\[VelC = \frac{Y\left(q_{op} + \Delta q\right) - Y\left(q_{op} - \Delta q\right)}{2 \Delta q}\]

\[DspC = \frac{Y\left(q_{op} + \Delta q\right) - Y\left(q_{op} - \Delta q\right)}{2 \Delta q}\]

\[D = \frac{Y\left(u_{op} + \Delta u\right) - Y\left(u_{op} - \Delta u\right)}{2 \Delta u}\]

\[D_d = \frac{Y\left(u_d_{op} + \Delta u_d\right) - Y\left(u_d_{op} - \Delta u_d\right)}{2 \Delta u_d}\]
Example
Campbell Diagram for OC3-Hywind
Current & Planned Work

- For tight coupling in **FAST** v8, introduce:
  - OP determination:
    - Static equilibrium (constant disp.)
    - Steady state (constant velocity)
    - Periodic steady state
    - With or without trim
  - Linearization
  - Across all coupled modules:
    - Structural dynamics
    - Aerodynamics
    - Hydrodynamics
    - Controls

\[
\begin{align*}
\Delta u^{(1)} & \rightarrow A^{(1)}, A^{DAC(1)}, B^{(1)}, \\
\Delta y^{(1)} & \rightarrow C^{(1)}, C^{DAC(1)}, D^{(1)} \\
\Delta u^{(2)} & \rightarrow A^{(2)}, A^{DAC(2)}, B^{(2)}, \\
\Delta y^{(2)} & \rightarrow C^{(2)}, C^{DAC(2)}, D^{(2)} \\
\vdots & \\
\Delta u^{(N)} & \rightarrow A^{(N)}, A^{DAC(N)}, B^{(N)}, \\
\Delta y^{(N)} & \rightarrow C^{(N)}, C^{DAC(N)}, D^{(N)}
\end{align*}
\]

Coupled Linear System
Questions?

Jason Jonkman, Ph.D.
+1 (303) 384 – 7026
jason.jonkman@nrel.gov

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