Linearization of FAST with AeroDyn & HydroDyn

CREW/NREL Wind Turbine Design Codes Workshop

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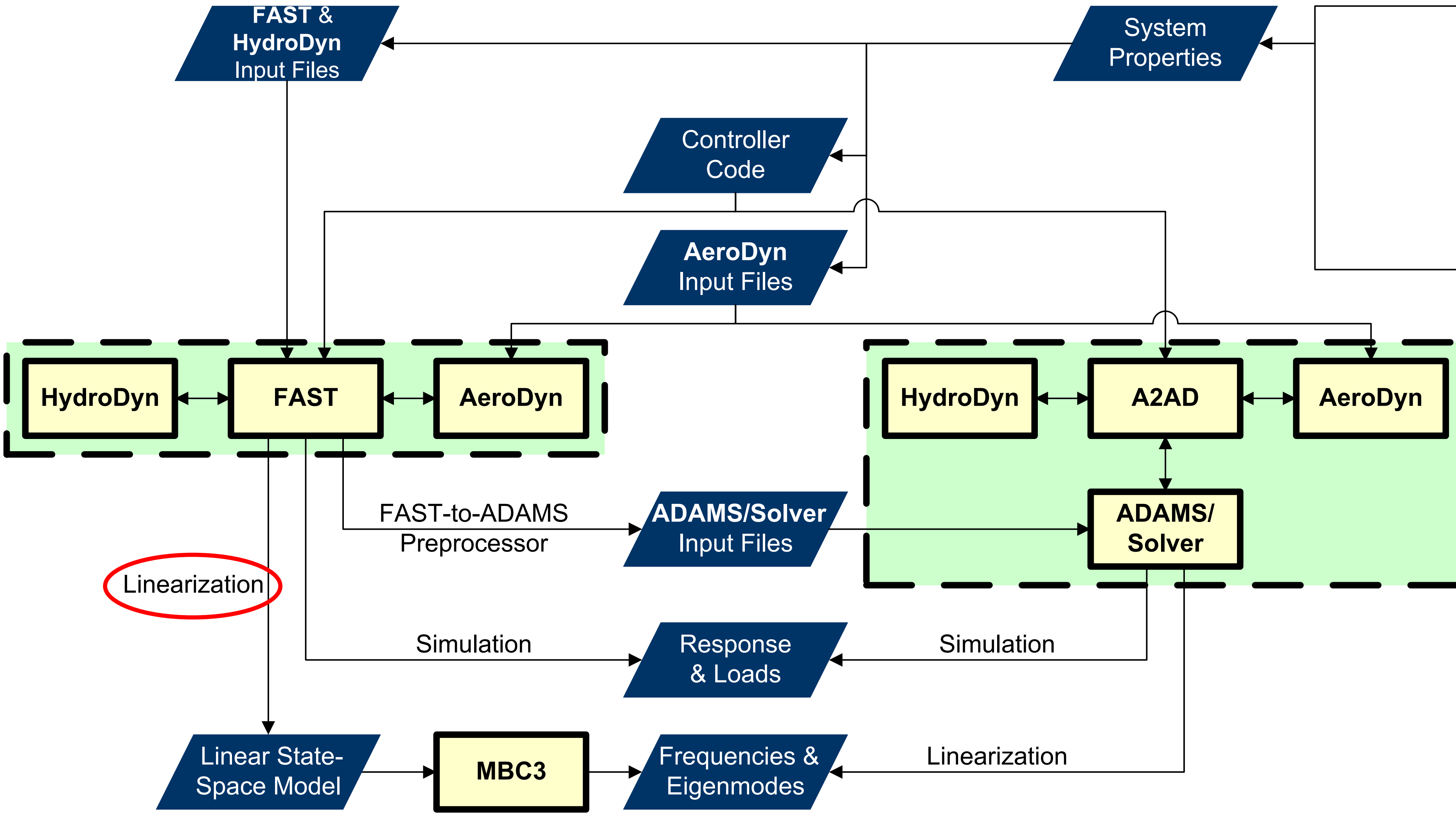
Outline

- Modes of Operation
- Background
- Calculating Operating Points
- Model Linearization:
  - 2^{nd} Order Model
  - 1^{st} Order Model
  - Numerics
- Example – Campbell Diagram for OC3-Hywind

Linearization
Modes of Operation



Linearization
Background

- Creates linear representation of nonlinear system model
- Applications:
  - Full-system modal analysis:
    - Frequencies
    - Damping
    - Mode shapes
    - Stationary or operational
  - Controls design:
    - Develop state-space representation of wind turbine “plant”
    - Includes control inputs, wind disturbances, & output
  - Stability analysis
- Linear model is only valid in the local vicinity of an operating point (OP):
  - Requires that one select or compute an OP before linearization
- When rotor is spinning, the linear system is periodic:
  - Azimuth-averaging averages-out the periodic effects
  - Multi-blade coordinate (MBC) expresses cumulative effect of blade dynamics in the fixed frame

Linearization

Calculating Operating Points

- Operating point choices:
  - Initial condition (user-selected) (watch out for nonstationary OP!)
  - Static equilibrium (parked or idling)
  - Periodic steady-state equilibrium (operational):
    - No trim: All controls fixed (fixed speed)
    - Trim: One control input varied to achieve desired rotor speed:
      - Nacelle yaw
      - Generator torque
      - Blade pitch
- Equilibrium solutions found through time-domain simulation:
  - Solution found within user-selected displacement & velocity tolerances
  - Solution can be sped up with optional compile-time feature to artificially increase system damping
  - Optional trim calculation is automated with a proportional feedback control law on rotor-speed error

Linearization

Model Linearization – 2nd Order Model

Nonlinear EoM:

$$M\left(\underline{q},\underline{u},t\right)\underline{\ddot{q}}+\underline{f}\left(\underline{q},\underline{\dot{q}},\underline{u},\underline{u}_d,t\right)=\underline{0}$$

Nonlinear Outputs:

$$\underline{OutData}=\underline{Y}\left(\underline{q},\underline{\dot{q}},\underline{u},\underline{u}_d,t\right)=Y_r\left(\underline{q},\underline{u},t\right)\underline{\ddot{q}}+\underline{Y}_t\left(\underline{q},\underline{\dot{q}},\underline{u},\underline{u}_d,t\right)$$

Perturbation of system variables:

$$\underline{q}=\underline{q}\big|_{op}+\underline{\Delta q}\qquad \underline{\dot{q}}=\underline{\dot{q}}\big|_{op}+\underline{\Delta\dot{q}}\qquad \underline{u}=\underline{u}\big|_{op}+\underline{\Delta u}\qquad \underline{u}_d=\underline{u}_d\big|_{op}+\underline{\Delta u}_d\quad\Rightarrow\quad \underline{\ddot{q}}=\underline{\ddot{q}}\big|_{op}+\underline{\Delta\ddot{q}}\qquad \underline{Y}=\underline{Y}\big|_{op}+\underline{\Delta Y}$$

Linear EoM:

$$M\,\underline{\Delta\ddot{q}}+C\,\underline{\Delta\dot{q}}+K\,\underline{\Delta q}=F\,\underline{\Delta u}+F_d\,\underline{\Delta u}_d$$

$$M=M\big|_{op}$$

$$C=\left.\frac{\partial\underline{f}}{\partial\underline{\dot{q}}}\right|_{op}$$

$$K=\left.\left[\frac{\partial M}{\partial\underline{q}}\underline{\ddot{q}}+\frac{\partial\underline{f}}{\partial\underline{q}}\right]\right|_{op}$$

$$F_d=-\left.\frac{\partial\underline{f}}{\partial\underline{u}_d}\right|_{op}$$

$$F=-\left.\left[\frac{\partial M}{\partial\underline{u}}\underline{\ddot{q}}+\frac{\partial\underline{f}}{\partial\underline{u}}\right]\right|_{op}$$

Linear Outputs:

$$\underline{\Delta Y}=VelC\,\underline{\Delta\dot{q}}+DspC\,\underline{\Delta q}+D\,\underline{\Delta u}+D_d\,\underline{\Delta u}_d$$

$$VelC=\left.\frac{\partial\underline{Y}}{\partial\underline{\dot{q}}}\right|_{op}=\left.\left[-Y_rM^{-1}C+\frac{\partial\underline{Y}_t}{\partial\underline{\dot{q}}}\right]\right|_{op}$$

$$DspC=\left.\frac{\partial\underline{Y}}{\partial\underline{q}}\right|_{op}=\left.\left[\frac{\partial Y_r}{\partial\underline{q}}\underline{\ddot{q}}-Y_rM^{-1}K+\frac{\partial\underline{Y}_t}{\partial\underline{q}}\right]\right|_{op}$$

$$D=\left.\frac{\partial\underline{Y}}{\partial\underline{u}}\right|_{op}=\left.\left[\frac{\partial Y_r}{\partial\underline{u}}\underline{\ddot{q}}+Y_rM^{-1}F+\frac{\partial\underline{Y}_t}{\partial\underline{u}}\right]\right|_{op}$$

$$D_d=\left.\frac{\partial\underline{Y}}{\partial\underline{u}_d}\right|_{op}=\left.\left[Y_rM^{-1}F_d+\frac{\partial\underline{Y}_t}{\partial\underline{u}_d}\right]\right|_{op}$$

(matrix sizes determined by enabled DOFs)

Linearization

Model Linearization – 1st Order Model

Conversion From 2nd Order to 1st Order:

$$\underline{x} = \begin{Bmatrix} \underline{\Delta q} \\ \underline{\Delta \dot{q}} \end{Bmatrix} \qquad \underline{\dot{x}} = \begin{Bmatrix} \underline{\Delta \dot{q}} \\ \underline{\Delta \ddot{q}} \end{Bmatrix} \qquad \underline{y} = \underline{\Delta Y}$$

Linear EoM & Outputs:

$$\underline{\dot{x}} = A\underline{x} + B\underline{\Delta u} + B_d\,\underline{\Delta u_d}$$

$$\underline{y} = C\underline{x} + D\underline{\Delta u} + D_d\,\underline{\Delta u_d}$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} \qquad B_d = \begin{bmatrix} 0 \\ M^{-1}F_d \end{bmatrix} \qquad C = \begin{bmatrix} DspC & VelC \end{bmatrix}$$

Linearization

Model Linearization – Numerics

- Linearization done numerically with central-difference method:
  - Perturbations made about OP
  - Perturbations are hard-coded:
    - Default perturbation: 2º
    - Can be changed at compile time

$$-M^{-1}C = \frac{\underline{\ddot{q}}\left(\left.\underline{\dot{q}}\right|_{op} + \underline{\Delta\dot{q}}\right) - \underline{\ddot{q}}\left(\left.\underline{\dot{q}}\right|_{op} - \underline{\Delta\dot{q}}\right)}{2\underline{\Delta\dot{q}}}$$

$$VelC = \frac{\underline{Y}\left(\left.\underline{\dot{q}}\right|_{op} + \underline{\Delta\dot{q}}\right) - \underline{Y}\left(\left.\underline{\dot{q}}\right|_{op} - \underline{\Delta\dot{q}}\right)}{2\underline{\Delta\dot{q}}}$$

$$-M^{-1}K = \frac{\underline{\ddot{q}}\left(\left.\underline{q}\right|_{op} + \underline{\Delta q}\right) - \underline{\ddot{q}}\left(\left.\underline{q}\right|_{op} - \underline{\Delta q}\right)}{2\underline{\Delta q}}$$

$$DspC = \frac{\underline{Y}\left(\left.\underline{q}\right|_{op} + \underline{\Delta q}\right) - \underline{Y}\left(\left.\underline{q}\right|_{op} - \underline{\Delta q}\right)}{2\underline{\Delta q}}$$

$$M^{-1}F = \frac{\underline{\ddot{q}}\left(\left.\underline{u}\right|_{op} + \underline{\Delta u}\right) - \underline{\ddot{q}}\left(\left.\underline{u}\right|_{op} - \underline{\Delta u}\right)}{2\underline{\Delta u}}$$

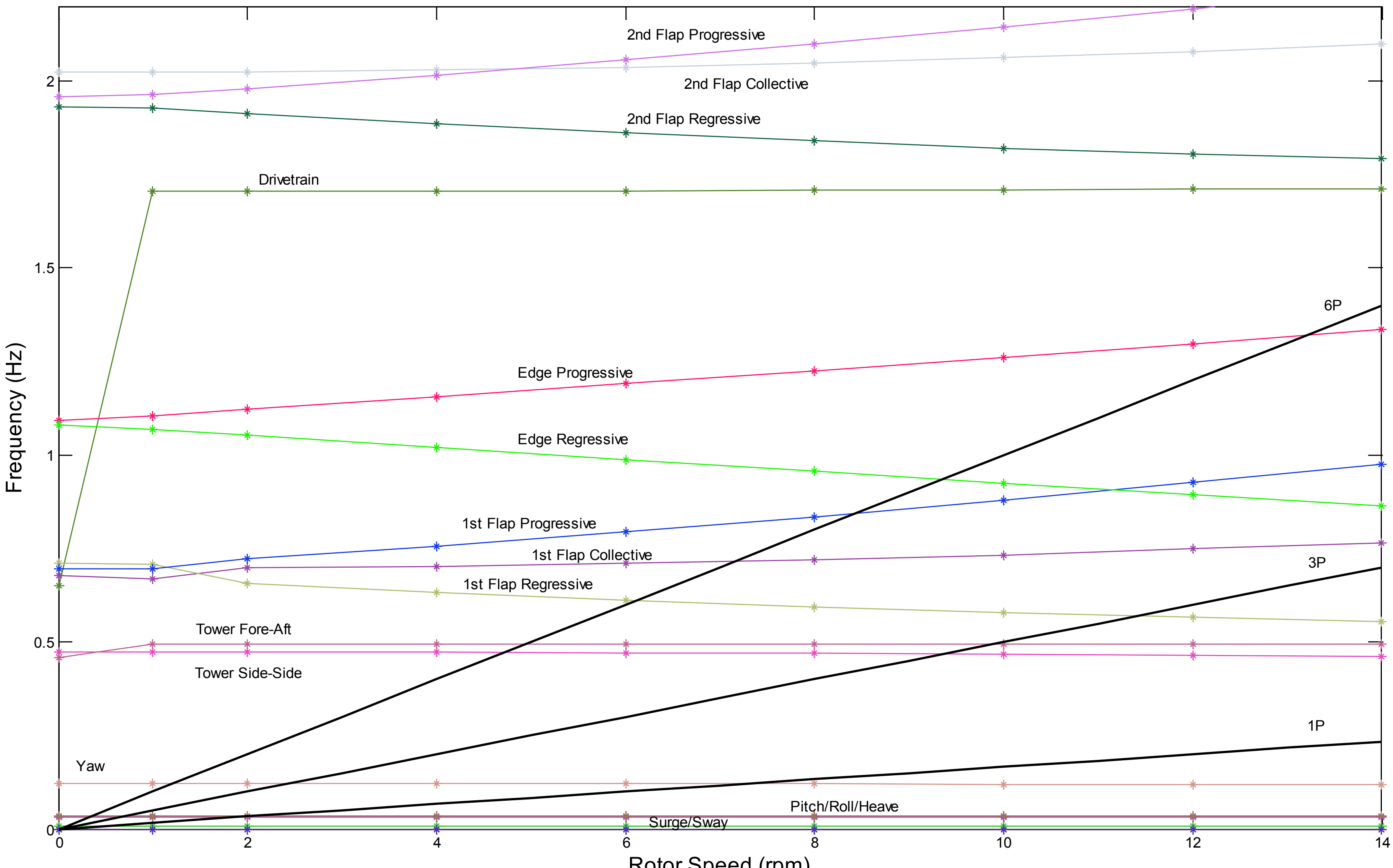
$$D = \frac{\underline{Y}\left(\left.\underline{u}\right|_{op} + \underline{\Delta u}\right) - \underline{Y}\left(\left.\underline{u}\right|_{op} - \underline{\Delta u}\right)}{2\underline{\Delta u}}$$

$$M^{-1}F_d = \frac{\underline{\ddot{q}}\left(\left.\underline{u_d}\right|_{op} + \underline{\Delta u_d}\right) - \underline{\ddot{q}}\left(\left.\underline{u_d}\right|_{op} - \underline{\Delta u_d}\right)}{2\underline{\Delta u_d}}$$

$$D_d = \frac{\underline{Y}\left(\left.\underline{u_d}\right|_{op} + \underline{\Delta u_d}\right) - \underline{Y}\left(\left.\underline{u_d}\right|_{op} - \underline{\Delta u_d}\right)}{2\underline{\Delta u_d}}$$

Linearization

Example – Campbell Diagram for OC3-Hywind



Questions?

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