Accuracy in wave load determination for offshore wind turbines

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1 Summary and conclusions

The paper summarises experiences with comparison of wave loads on a 6 m monopile at water depth h=25 m. The wave loads are determined by various theories and scaled physical model tests.

It is concluded that only the Boussinesq/Lundgren method may provide an accurate wave load time-series description. Anyhow, the stream function/Morison method allows for a very accurate non-linear single wave load calculation which is an important and recognized reference point. The simulations show a very high correlation between the single wave force calculated by stream function/Morison and the 0.1 % event from Boussinesq/Lundgren time-series.

The linear methods provide an inaccurate estimate, where the deviations increase with the ratio of the significant wave height to the water depth.

The experiences with physical model tests show that a certain scatter of results should be expected, especially for the moments. For the well investigated structure and wave situation in the present paper the numerical simulations are more precise. However for more complex situations physical model tests are very important for verification.

2 Introduction

The paper summarises the general experiences with comparison of wave forces/moments determined by various theories and corresponding model test results. Much of the experiences have been obtained in connection to the studies for offshore wind projects carried out by Energi E2 including Middelgrunden, Nysted, Borkum Riffgrund, Scarweather Sands, London Array plus projects carried out by Carl Bro as separately including Lillgrund and Clogher Head.

The methods investigated in the present paper are summarised in table 1.

Method name	Wave type	Wave model	Force model
Boussinesq/Lundgren	Irregular waves	Boussinesq	Lundgren
Boussinesq/Morison	Irregular waves	Boussinesq	Morison
JONSWAP/Lundgren	Irregular waves	Stokes 1. order	Lundgren
JONSWAP/Morison	Irregular waves	Stokes 1. order	Morison
Stream function/Lundgren	Regular waves	Stream function (30)	Lundgren

Table 1: The table summarises the wave force calculation methods used in the present paper.

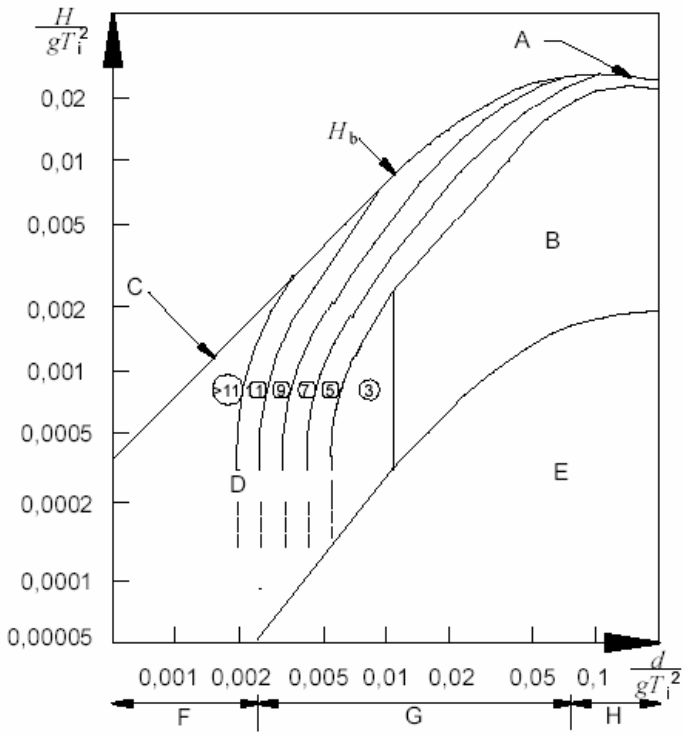
All calculations with irregular waves in the present paper are based on the JONSWAP spectrum with peak enhancement factor γ=3.3. In the methods based on the Stokes 1st order the wave surface elevation time-series are generated directly from the JONSWAP spectrum by means of Inverse Fourier Transform (IFFT). In the methods based on Boussinesq simulations, the wave surface elevation and kinematics are derived with a high order Boussinesq wave model with an input (lateral boundary condition) corresponding to the abovementioned method using the JONSWAP spectrum and IFFT.

In the single wave calculations the expected maximum (0.1 % event) wave height are determined by the methods described by Battjes, cf. ref [1]. All stream function calculations in the present paper are performed with 30th order precision.

An example of determination of extreme loads performed with the Boussinesq/Lundgren method is illustrated, including verification with the stream function/Morison method.

3 Wave models

In numerical simulations it is important to use a suitable wave theory. The validity ranges of regular wave theories are summarised in figure 1.



Key

A	Deep water breaking limit H /λ = 0,14	E	Linear / Airy or 3rd order Stream Function
B	Stokes' 5th order, New Wave or 3rd order Stream Function	F	Shallow water
C	Shallow water breaking limit H/ = 0,78	G	Intermediate depth
D	Stream Function (showing order number)	H	Deep water

Figure 1: Validity of various regular wave theories (ref [2]). In this figure d is the depth.

Close to the breaking limit only higher order stream function methods are valid except for the deep water case where high order Stokes are valid as well.

In the case of irregular waves the starting point or input to any model is usually based on Stokes 1st order (Linear/Airy) theory. Either this is used alone or as input for a wave model. The only wave model for irregular waves investigated in the present paper is a high order Boussinesq model, which is very precise, cf. ref [8]-[10].

Figure 2 shows an example of wave surface profiles at intermediate water depth computed with the wave model used in the present study.

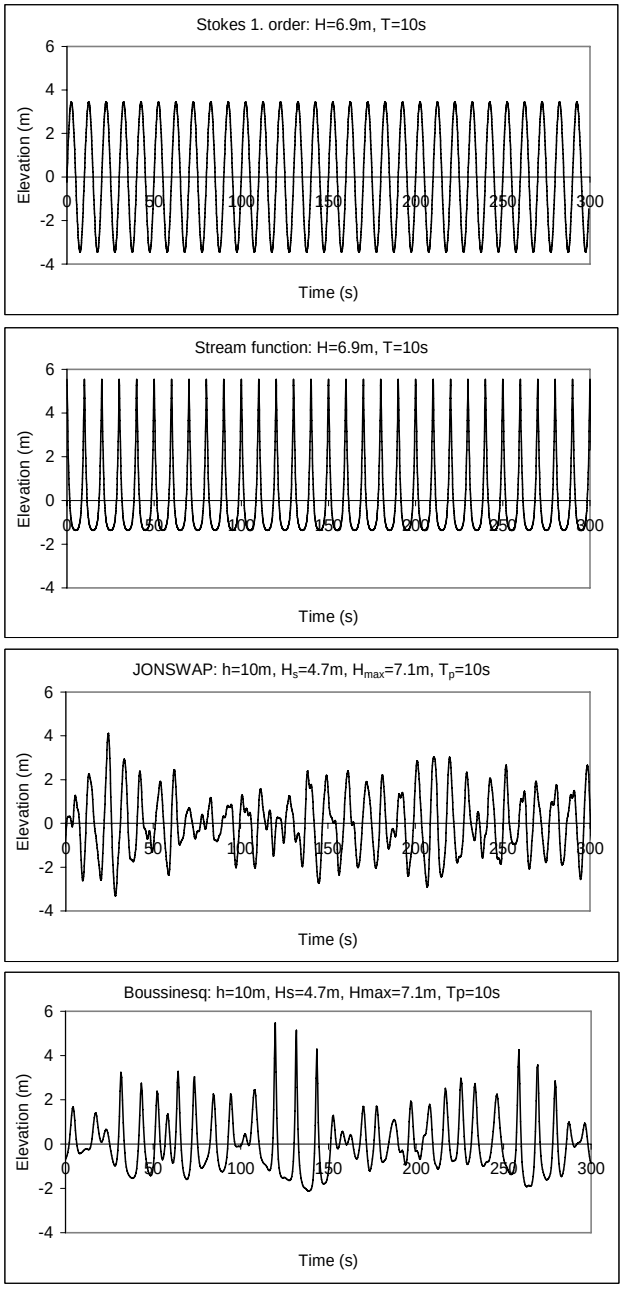


Figure 2: Illustration of surface elevation profile computed with different wave models.

It is clear from figure 2 that the Boussinesq model has advantages over the other models due to the combination of high order waves and irregular waves, and the wave surface profile has some similarities with the Stream function surface profile. An example of the wave kinematics computed with the Boussinesq model is illustrated in figure 3.

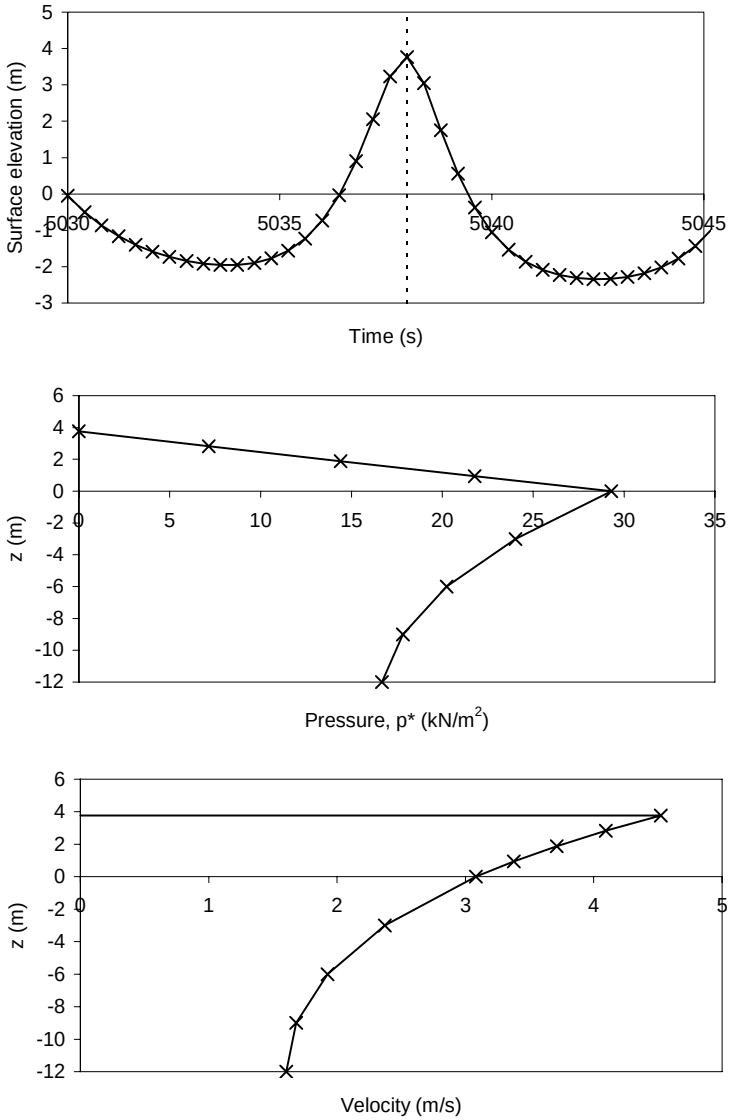


Figure 3: Illustration of kinematics in non-linear wave

Figure 4 and 5 compare the velocity and pressure profiles calculated with the Boussinesq model, stream function theory and directly from the JONSWAP spectrum using Stokes 1st order wave theory. We see a clear difference between the Boussinesq and JONSWAP based results. Since the Boussinesq method is valid and the Stokes 1st order is not valid, the JONSWAP/Stokes method seems to give imprecise description of the velocity and pressure.

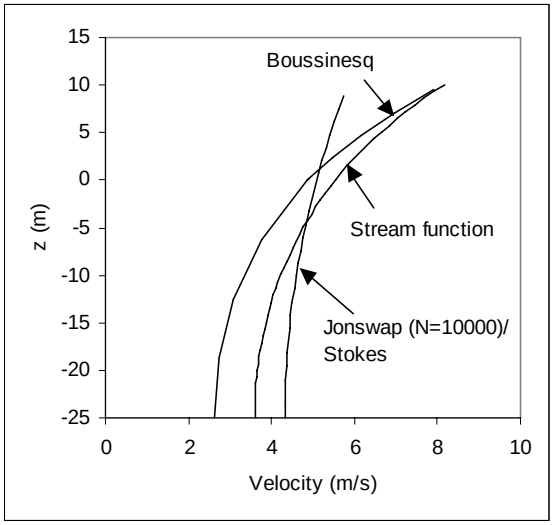


Figure 4: The figure shows the velocity distribution over the depth at a single time-step. Velocities from the 3 wave models discussed in the present paper are compared.

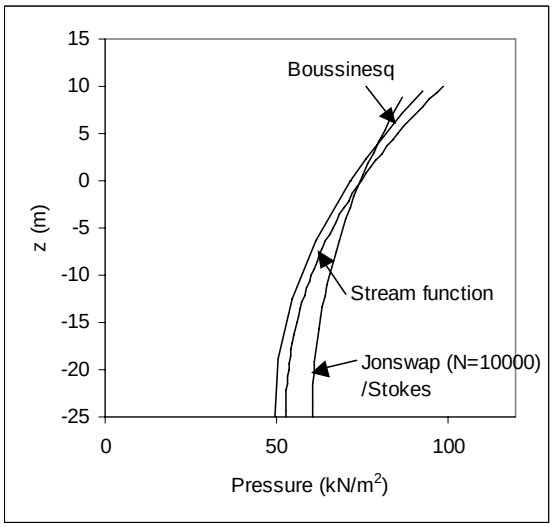


Figure 5: The figure shows the pressure distribution over the depth at a single time-step. Pressures from the 3 wave models discussed in the present paper are compared.

When a wave model is chosen, this enables the computation of time-series of undisturbed (no structure) kinematics in any point over the depth, i.e. the surface elevation, pressure, acceleration and velocity. The presence of a structure however influences the pressure field and hence a force model is needed to calculate the forces on a structure, unless the chosen wave model includes the opportunity to include structures. In the latter case the force is the pressure integrated over the surface of the structure.

4 Force model

In the present paper, two force models are used namely the Morison equation and the Lundgren method.

The Morison equation is an estimate of the forces given the velocity and acceleration at the centre of the structure, i.e. both the undisturbed pressure at the structure surface and the pressure due to the presence of the structure are estimated from the kinematics at the structure centre. This makes it easy and convenient to calculate the forces with the Morison equation. In the Morison equation the force coefficients are set to C_d = 1.0 and C_M = 2.0 in the present work.

The Lundgren method is an estimate of the forces given the pressure at a distance 0.8 D from the centre of the structure and the velocity at the centre of the structure. This enables a better approximation to the inertia force in non-linear waves by means of account to a non-constant acceleration over the effective volume of the structure and inclusion of effect from finite wave heights. For a constant acceleration of a symmetrical wave the Lundgren method results in the same inertia force as by applying C_M = 2.0 in the Morison equation. Cf. ref [4], [5] and [7]

5 Comparison between the different design methodologies

The present section describes the verification of results from wave force calculations for a monopile with diameter D=6 m, placed at water depth h=25 m. The main calculations have been performed with the Boussinesq/Lundgren method, and stream function results are used for verification of the results for the highest waves. For comparison results of the Jonswap/Morison method using Wheeler Stretching are presented as well.

The Boussinesq/Lundgren method is desirable because of the need of force time-series. The stream function method is a single wave theory, so this method can only be used the check the results. On the other hand this single wave case represents an accepted offshore extreme wave design case.

The JONSWAP based methods are found to be insufficient, as they cannot describe forces from steeper waves at relatively shallow waters.

The validity of the Morison equation for irregular waves will be discussed.

5.1 Boussinesq/Lundgren results

Prof. Per Madsen and Dr. Harry Bingham have computed full time-series of kinematics with their recently developed Boussinesq time-domain model valid for much deeper water than commonly used Boussinesq models, cf. ref [8]-[10]. The Boussinesq model calculates the evolution of fully non-linear steep and irregular waves in a pseudo 2-D model. The Boussinesq data applied is computed with at bottom slope of approximately 1%. The deep-water wave input for the model is generated from a JONSWAP spectrum with the ratio of deep-water wave height to deep-water wavelength S_{op} = 0.030. Time-series of approximately 1000 waves have been computed. The Lundgren method will be applied on the Boussinesq kinematics to calculate time-series of the wave force. These time-series are analyzed and presented mainly in terms of exceedance probability distributions, spectra and statistical properties. The statistical interpretations are used for calculating the maximum wave loads of 1,000 waves 10,000 waves and 100,000 waves.

5.1.1 Wave forces

The wave forces will be analysed in terms of the exceedance distributions by the following model, see ref. [4], [5] and [7].

$$F = k \cdot \left(\frac{1 - \exp(\alpha \cdot \log10(P(F > x)))}{\alpha} \right) + offset$$

Where the k, α and offset are arbitrary constants.

Linear interpretations as illustrated in table 2 are carried out, with the following notation:

$k = k_1 \cdot H_s + k_2$

$\alpha = \alpha_1 \cdot H_s + \alpha_2$

$offset = offset_1 \cdot H_s + offset_2$

k_1	220
k_2	240
α_1	-0.03
α_2	0.7
$offset_1$	55
$offset_2$	0

Table 2: The table shows parameter interpretations for a 6 m monopile at 25 m water depth.

From the parameters given in table 2, wave force distributions for H_s = 1, H_s = 2, … , H_s = 9 m are given in figure 6. The results for H_s = 4-8 m are shown in table 3.

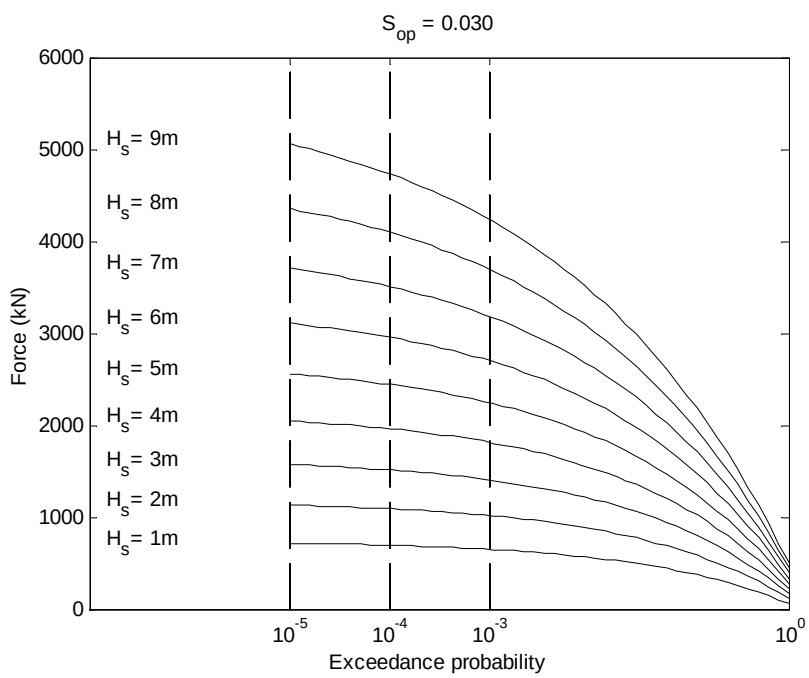


Figure 6: Generalized force exceedance distributions calculated with the Lundgren/Boussinesq method

Given the wave statistics for the wind turbine site (example figure 7) the wave load distribution and wave height distribution can be integrated to give the design wave load, see example figure 8.

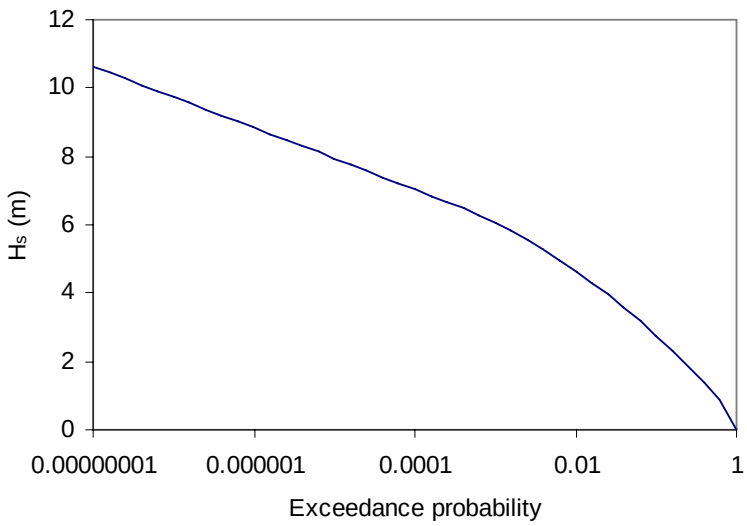


Figure 7: Example of wave height statistics.

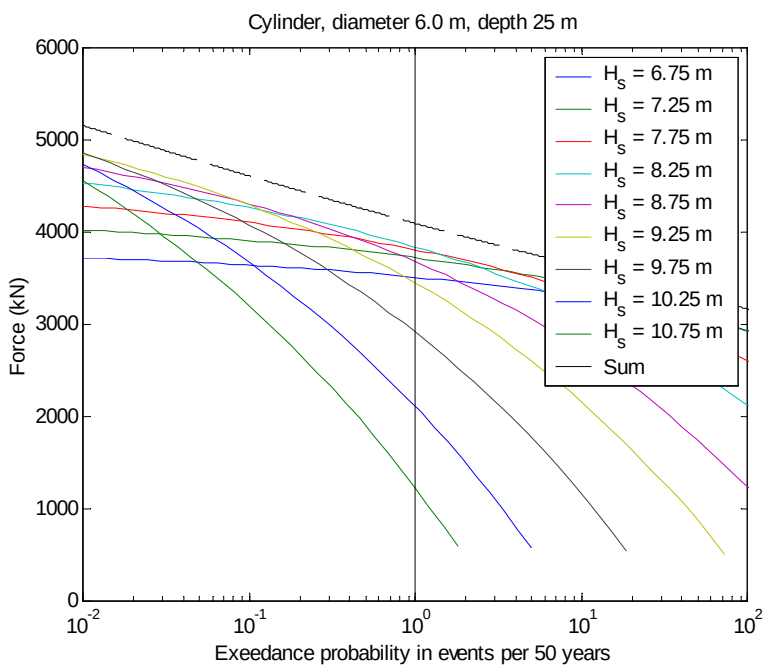


Figure 8: Integration of the wave force distributions in figure 6 and the assumed wave height distribution in figure 7.

H_s (m)	4	5	6	7	8
$F_{0.1\%}$	1813	2244	2701	3184	3696
$F_{0.01\%}$	1962	2442	2956	3507	4099
$F_{0.001\%}$	2046	2557	3108	3706	4354

Table 3: Interpreted results of the maximum wave force on a 6m monopile exposed to 1,000 waves, 10,000 waves and 100,000 waves at 25 m water depth. The results are calculated with the Boussinesq/Lundgren method.

5.1.2 Moment arm (seabed level)

The seabed level moment arm is interpreted from the exceedance probability distributions of forces and moments at seabed level. This is done distribution-wise for the 10 largest values of force and the 10 largest values of moment. Note that the moment and force paired in this way, does generally not occur simultaneously. Stream function results are shown as well. The moment lever arm calculated by the stream function/Morison approach is slightly higher than those calculated by the Boussinesq/Lundgren model. This is supposed to be a consequence of the difference in a regular wave and irregular wave representation.

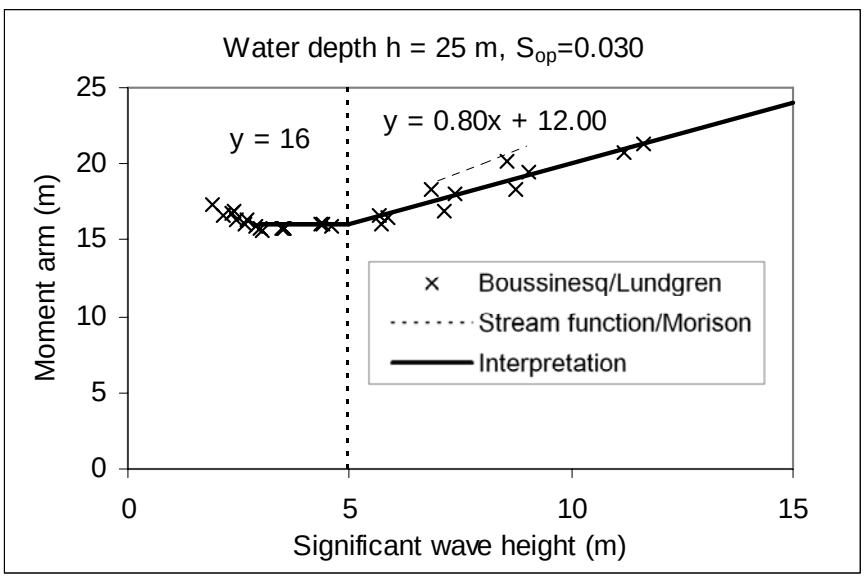


Figure 9: Moment lever arm for a 6 m monopile at 25 m water interpreted from distributions.

5.2 Boussinesq versus stream function

With respect to the forces there is very little difference between the results from the maximum of the Boussinesq time series and the stream function single wave result, see figure 10.

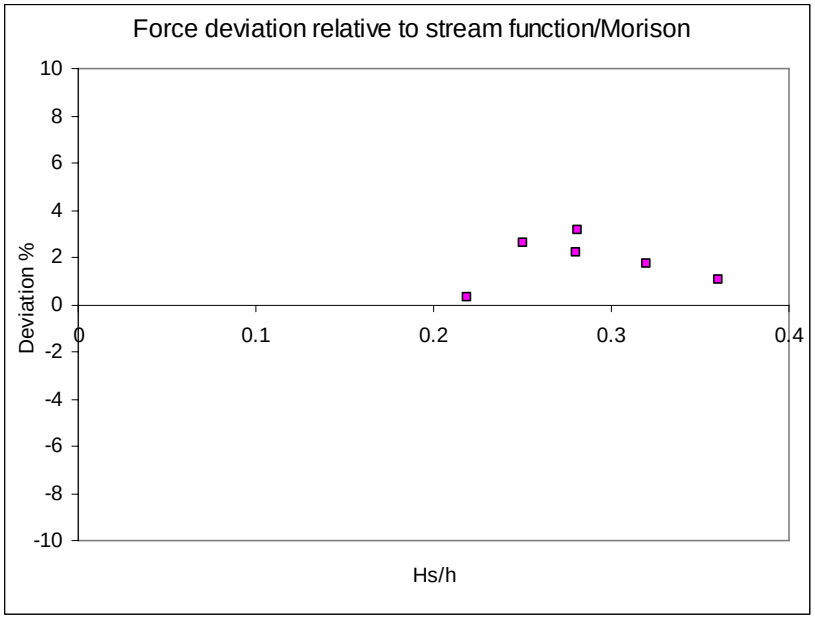


Figure 10: Force results from Boussinesq/Lundgren wave load calculations for a 6 m monopile performed at a water depth of 25 m for wave steepness 0.03. The results are related to stream function/Morison results. The basis of the calculations is water depths 25 m and 32 m and significant wave heights 7 m, 8 m and 9 m.

With respect to the moments there is a somewhat larger difference between the results from the maximum of the Boussinesq time series and the stream function single wave result varying between 0-10 % for H_s/h = 0.2-0.4, see Figure 11. The difference is largest at the largest wave heights. It is deemed that the Boussinesq simulation represents a better estimate to the real wave conditions than the single wave solution because a lower lever arm is to be expected at the larger waves in a irregular wave train because these are associated with a lower water level due to an increase in the radiation stress proportional to the wave height squared.

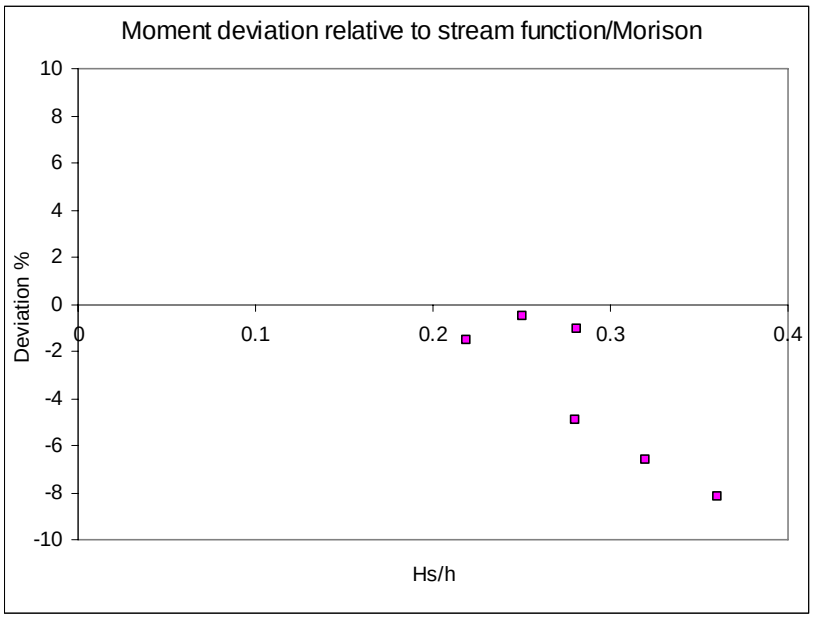


Figure 11: Moment results from Boussinesq/Lundgren wave load calculations for a 6 m monopile performed at a water depth of 25 m for wave steepness 0.03. The results are related to stream function/Morison results. The basis of the calculations is water depths 25 m and 32 m and significant wave heights 7 m, 8 m and 9 m.

5.3 Model tests versus calculated results

The comparison is based on physical model tests in scale 1:50 carried out at Aalborg University, see figure 12.

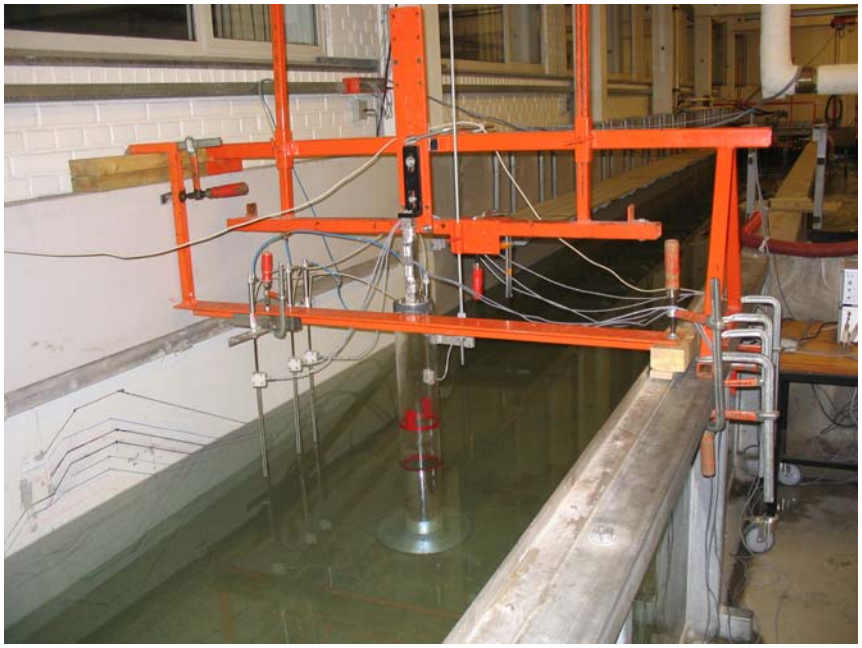


Figure 12: Illustration from force model tests

Generally there have been quite some scatter on the results from the physical model tests, and accordingly some verification tests have been performed and some of the force results had to be smoothed. The results for the 6 m monopile are shown in figure 13.

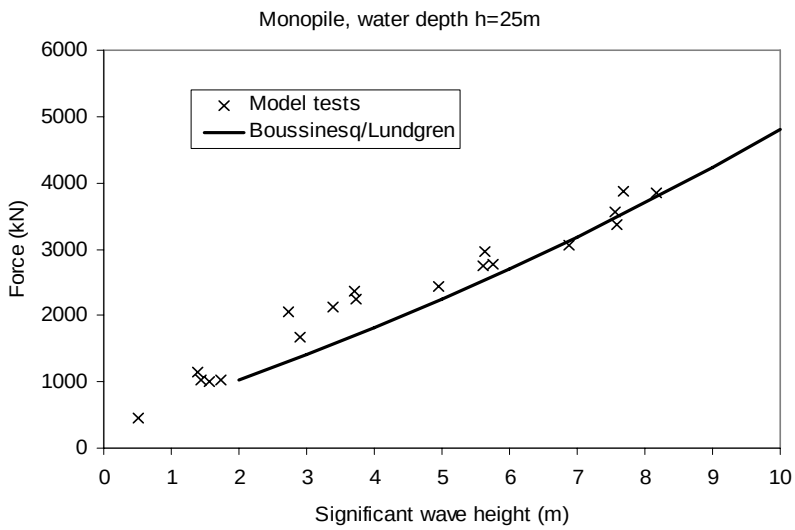


Figure 13: Comparison of force results with Boussinesq/Lundgren wave force calculations

The results for a 6 m monopile at about h=25m water exposed to a wave height of H_s=5.3-5.8 m are summarized in Table 4.

	H_s (m)	$F_{0.1\%}$ (MN)	$M_{0.1\%}$ (MNm)
Physical model tests	5.8	2.76	61.3
	5.3	2.52	57.5
Boussinesq/Lundgren	5.8	2.61	43.4
	5.3	2.38	38.7

Table 4: The table shows results for monopile at water depth h=25 m.

Further problems with model tests include:

- Scale effect (to the drag forces)
- Practical problems of carrying out model test with a small smooth slope due to length limitations
- High frequency components are not modelled correctly
- Measured forces are in practice deflections of an elastic supported structure including a resonance frequency

5.4 Verification from single wave calculations

For verification of the wave force time-series and interpretations, wave forces are calculated using the stream function theory. As this is a single wave approach, we need to choose design wave parameters for the calculation. To do this we will take advantage of the Battjes & Groenindijk wave distributions described in ref [1]. The data are summarized in table 5.

H_s (m)	7	8	9
$\sqrt{m_0}$	1.75	2.00	2.25
H_{rms} (m)	5.10	5.90	6.71
$\tilde{H}_{tr}$	2.00	1.73	1.52
$\tilde{H}_{0.1\%}$	2.33	2.20	2.09
$H_{0.1\%}$ (m)	11.9	13.0	14.0
$H_{0.1\%}/H_s$	1.70	1.62	1.56

Table 5: Maximum wave height calculated from the Battjes & Groenendijk distributions

The peak period is calculated from the two definitions below:

$$S_{op} = \frac{H_{s,0}}{L_{op}} \qquad \text{and} \qquad L_{op} = \frac{g \cdot T_p^{\,2}}{2 \cdot \pi}$$

Which yields:

$$T_p = \sqrt{\frac{2 \cdot \pi \cdot H_s}{g \cdot S_{op}}}$$

The peak period will be used as the period for the calculations because it is been verified that this results in a close correlation between the statistical force/moment (at probability 0.1%) and the forces calculated from the stream function theory based on a realistic maximum wave (0.1%).

The results from the stream function/Morison calculations are presented in table 6, where the results are compared with similar results from the Boussinesq/Lundgren and JONSWAP/Morison results.

	Wave loads		Deviation from Streamfunction/Morison	
	$F_{0.1\%}$ (MN)	$M_{0.1\%}$ (MNm)	$F_{0.1\%}$ (%)	$M_{0.1\%}$ (%)
Stream function/Morison	3.63	72.8	-	-
Boussinesq/Lundgren	3.70	68.0	1.7	-6.6
JONSWAP/Morison	3.51	58.6	-3.4	-19.5

Table 6: Wave loads for a 6m monopile placed at 25 m water, for H_s = 8.0 m, H_{max}=13.0 m and T_p=13.1 s. Stream function results (single wave) are compared to interpretations of the Boussinesq/Lundgren and JONSWAP/Morison methods.

5.5 Morison equation versus Lundgren method

In the stream function calculations above, the Morison equation is applied. The stream function/Lundgren and stream function/Morison method have been compared for several calculations on large waves on water depths about 25m. In both methods the inertia coefficient is chosen as C_M=2. In all cases the deviations were less than 1% on the force.

The two methods have been compared for smaller wave heights, and here we found that the Morison equation yields higher forces than the Lundgren method, especially for small waves (D/L<2). This is because the Morison equation is based on the acceleration and not directly on the pressure, which basically means that the Morison equation interpret the pressure difference from the acceleration, which introduces an error in this case.

In calculations with a substantial ratio between the significant wave height and the water depth, the asymmetry of the wave is strong enough for the highest waves to introduce a difference in the two predictions, which indicate that the acceleration approach to predict the pressure difference used in the Morison equation yields a higher pressure difference than the actual pressure difference, which means that an error is introduced. There is limited difference in the force spectra except on higher frequencies where the Lundgren method acts as a relevant filter not being present in the other model, see figure 14.

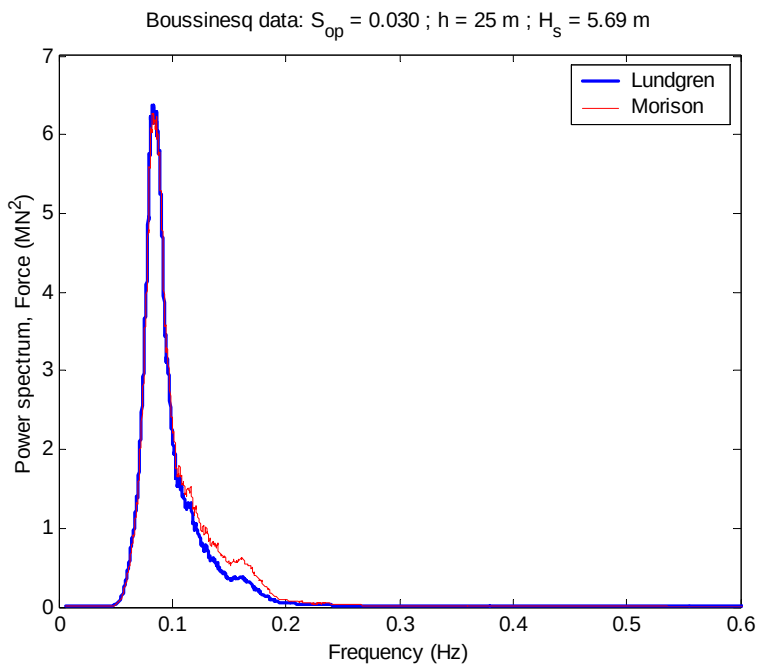


Figure 14: Example of comparison of wave force spectra calculated with the Lundgren method and the Morison equation. Both are based on the same Boussinesq data.

For the wave load exceedance distributions there are larger deviations, see figure 15. The Lundgren method yields smaller wave load than the Morison equation when irregular Boussinesq waves are used.

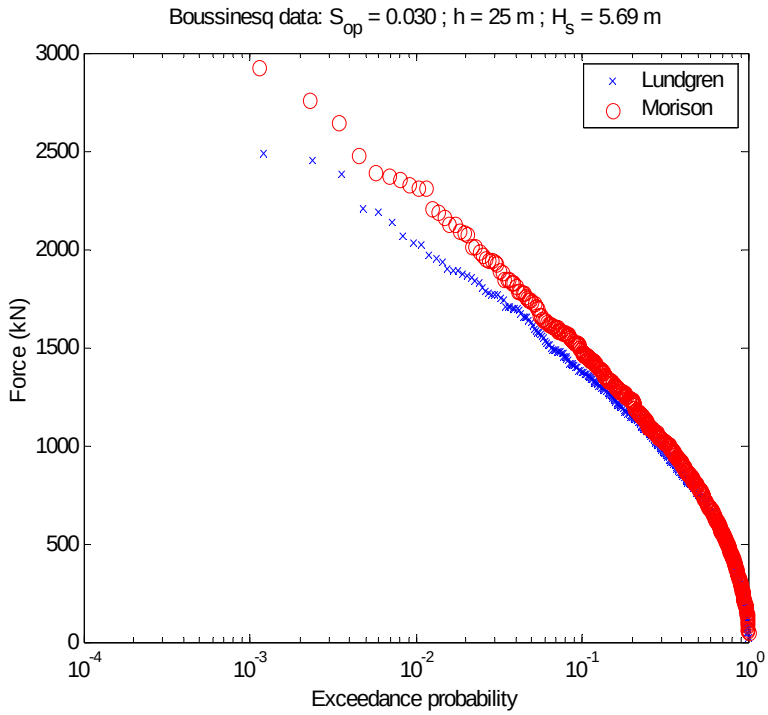


Figure 15: Example of comparison of wave force distributions calculated with the Lundgren method and the Morison equation. Both are based on the same Boussinesq data.

5.6 JONSWAP / Morison results

In Table 6, results predicted with the JONSWAP/Morison method have already been introduced. The results have been produced by generation of 20 time-series of the surface elevation. The one of the 20 time-series that has a wave height distribution that best fits the Rayleigh distribution is used for calculating the force on the structure of for about 1000 wave periods. The calculation of forces re-introduces a scatter in the results, and this scatter is evened out by interpretation of the exceedance distributions of the force with the same method as used earlier. The curvature factor α are reused from the Lundgren/Boussinesq results.

The results produced in this test show that the force distributions and spectra for the high waves on water depth about 25m are close to those predicted with the Boussinesq/Lundgren method. For higher and longer waves and on more shallow water the force is under determined by the JONSWAP method, because the Stokes method is not valid in this range. The method tends to over predict the shorter waves due to the use of the Morison equation.

5.7 Summary

The results produced with the Boussinesq/Lundgren method have been compared with results from other methods, based on the stream function theory, the Morison equation and the direct JONSWAP/Stokes method. The Boussinesq/Lundgren is found to yield the best overall results, when time-series are to be calculated. Compared to the stream function results this method yields reasonable results.

A comparison of the results obtained for the 6 m monopole at 25 m water depth has been illustrated (for wave steepness 0.03) at figure 16 and 17. The main results are extracted in Table 7.

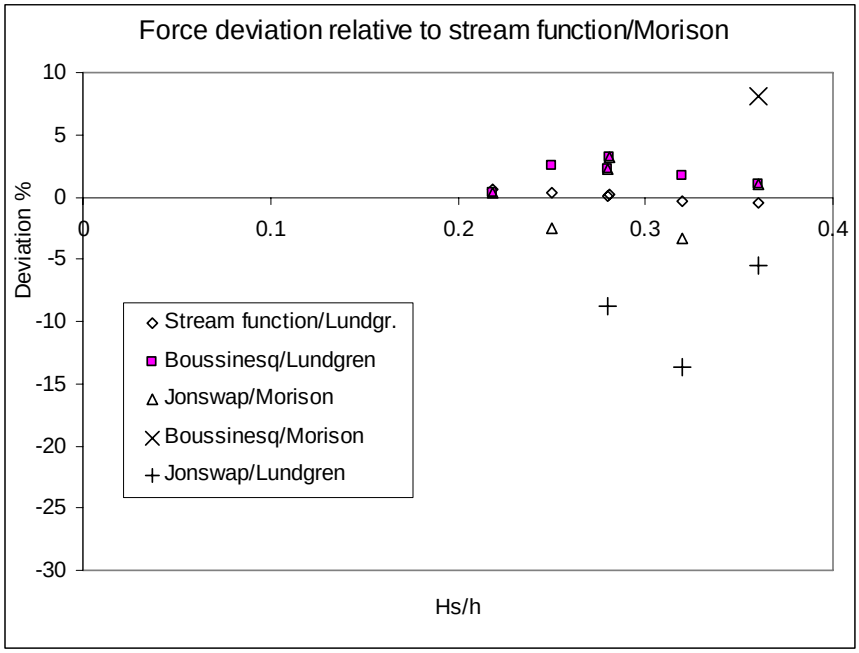


Figure 16: Force results from wave load calculations for a 6 m monopile performed at a water depth of h=25 m for wave steepness 0.03. The results are related to stream function/Morison results. The basis of the calculations is water depths 25 m and 32 m and significant wave heights 7 m, 8 m and 9 m.

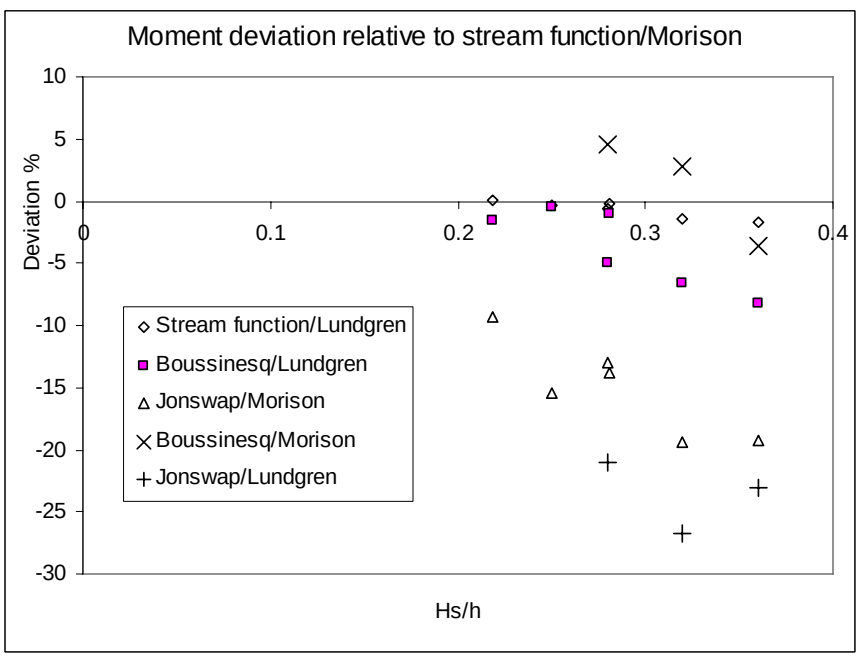


Figure 17: Moment results from wave load calculations for a 6 m monopile performed at 25 m water depth for wave steepness S_{op} = 0.03. The results are related to stream function/Morison results. The results are related to stream function/Morison results. The basis of the calculations is water depths 25 m and 32 m and significant wave heights 7 m, 8 m and 9 m.

Stream function/ Lundgren	$F_{0.1\%}$	OK
	$M_{0.1\%}$	OK
Boussinesq/Lundgren	$F_{0.1\%}$	OK
	$M_{0.1\%}$	>0.3
Boussinesq/Morison	$F_{0.1\%}$	> 0.15
	$M_{0.1\%}$	OK
Jonswap/Morison	$F_{0.1\%}$	OK
	$M_{0.1\%}$	> 0.15
Jonswap/Lundgren	$F_{0.1\%}$	> 0.22
	$M_{0.1\%}$	> 0.15

Table 7: Hs/h ratios with forces deviations larger than 5 % (see figure 16 and 17).

From Table 7 it may be concluded that only the Boussinesq/Lundgren model includes a reasonable accuracy when time-series are to be produced. The model includes a slightly lower moment (5-10 % lower than the stream function theory) for Hs/h exceeding 0.3. But the reality might be that the Boussinesq/Lundgren model is more accurate estimate for the irregular wave case than the reference stream function/Morison single wave theory because the time series simulations include a physically correct reduction in the local water level during the largest waves.

With respect to the previous usually applied Jonswap/Morison method the forces are usually (at least for Hs/h < 0.2) of correct order of magnitude but the moments are highly underestimated (for nearly all Hs/h > 0.15).

There is no reason to consider applying the Boussinesq/Morison, which overestimate the forces and specially the moments or Jonswap/Lundgren, which underestimate the forces and specially the moments.

6 References

[1] Battjes, J.A. and Groenendijk, H.W (2000): Wave height distributions on shallow foreshores. Coastal eng. 40 (2000) 161-182.

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