Efficient uncertainty propagation through an aero-elastic wind turbine model

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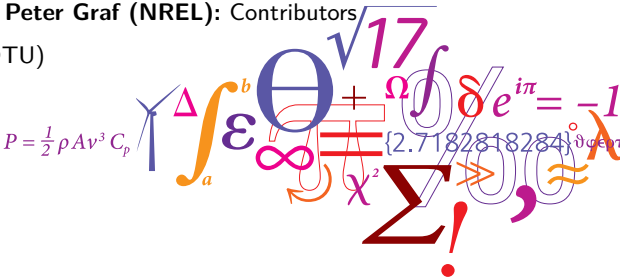


DTU Wind Energy
Department of Wind Energy

Outline

1. (A very short) Introduction to Polynomial Chaos Expansions
2. Polynomial Chaos Expansions as wind turbine aero-elastic model surrogates
3. Results
4. Future Work

Uncertainty propagation problem

Input variables
$\mathbf{x} \in \mathbb{R}^M$

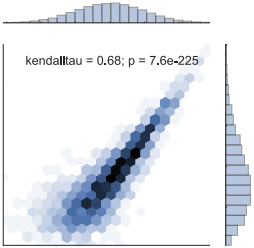
→ Model $\mathcal{M}(\mathbf{x})$ →

Output variables
$\mathbf{y} = \mathcal{M}(\mathbf{x}) \in \mathbb{R}^L$

Stochastic inputs
$\mathrm{PDF}(\mathbf{x})$

→ Model $\mathcal{M}(\mathbf{x})$ →

Stochastic outputs
$\mathrm{PDF}(\mathbf{y})$?



?

Monte-Carlo simulation

- Sample the joint input distribution:

$$\mathbf{x}_i \sim \mathrm{PDF}(\mathbf{x})$$

- Input sample can be generated using variance reduction methods such as: Latin Hypercube sampling (LHS), Halton or Hammersley sequences.
- Obtain a response sample by evaluating the model in each input realization:

$$\mathbf{y}_i = \mathcal{M}(\mathbf{x}_i)$$

- **Pros:** Very robust and easy to implement and parallelize
- **Cons:** Convergence is slow ($\propto N^{-1/2}$)

Polynomial Chaos expansion

- Build a polynomial surrogate of the model:

$$y(\mathbf{x}) \approx \hat{y}(\mathbf{x}) = \sum_{l=0}^{N_c-1} c_l \, \phi_l(\mathbf{x})$$

- A polynomial basis, $\phi_l(\mathbf{x})$, is built with respect to $\mathrm{PDF}(\mathbf{x})$

Distribution	Polynomial Family
Uniform	Legendre
Normal	Hermite
Exponential	Laguerre

- The model is evaluated, $\mathbf{y}_i = \mathcal{M}(\mathbf{x}_i)$, and *projected/fitted* to the polynomial basis.
- A MC sample is generated using the surrogate.
- Sensitivity analysis is estimated from the MC sample Saltelli et al. [Saltelli 2010].
- **Pros:** Convergence is fast ($\propto N^{-m}$, $m > 1$, m is problem dependent)
- **Cons:** How to define the order of the polynomials in each variable? How to avoid over fitting? Hot to avoid oscillations?

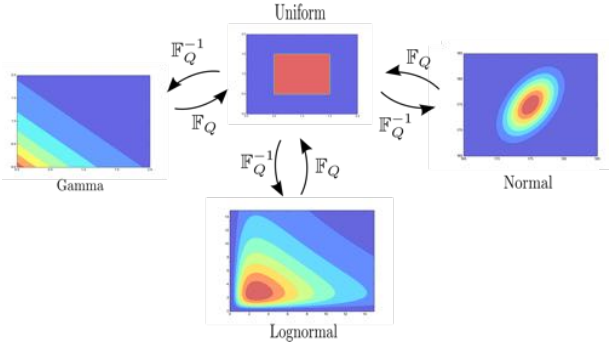
How to deal with correlated inputs?

Rosenblatt Transformation [Rosenblatt 1952]

- Transforms the correlated input variables ($\mathbf{x}$) into a multi-dimensional uncorrelated uniform space ($\mathbf{w}$). Solve the propagation problem in the uncorrelated space:

$$\mathbf{y}(\mathbf{x}) = \mathbf{y}(\mathbb{F}_Q^{-1}(\mathbf{w})) \approx \hat{\mathbf{y}}(\mathbf{w}) = \sum_{l=0}^{N_c-1} c_l \, \boldsymbol{\phi}_l(\mathbf{w})$$

- Rosenblatt transformation consists in using the inverse of the CDF of each variable in sequence. Chaospy includes this transformation [Feinberg 2015]. Graph reproduced from Chaospy tutorials.



Methods to find the coefficients c_l

Semi-Spectral projection (quadrature integration)

- Use a quadrature rule to approximate the integrals (nodes, x_i and weights ω_i). Gaussian quadrature is widely used.

$$c_l = \langle y, \boldsymbol{\phi}_l \rangle = \int y(x)\, \boldsymbol{\phi}_l(x)\, \mathrm{PDF}(x)\, dx \approx \sum_{i=0}^{N} \omega_i\, y(x_i)\, \boldsymbol{\phi}_l(x_i)$$

- **Pros:** Very good for low number of dimensions
- **Cons:** Unstable for heavy tailed PDFs. Quadrature rules fail with most correlated variables

Point collocation (polynomial fit)

- Generate a small sample and fit the polynomial basis using Least squares or some other optimization method (e.g. LAR, LASSO).
- **Pros:** Very robust. Optimization algorithms are design to handle large number of dimensions (sparsity) and correlated inputs.
- **Cons:** Not as efficient as semi-spectral collocation.

PCE of complex cases

Variable transformations steps

Correlated inputs $\mathrm{PDF}(\mathbf{x})$ → Rosenblatt transformation → Uniform **independent** $\mathrm{PDF}(\mathbf{w})$ → PCE $\mathbf{z}(\mathbf{x}) \approx \hat{\mathbf{z}}(\mathbf{w})$ → Overlimited outputs $\mathrm{PDF}(\mathbf{z})$ → Logistic transformation → **Correlated** outputs $\mathrm{PDF}(\mathbf{y})$

Rosenblatt transformation

- Used to decorrelate the variables
- Variables can be transformed to independent Uniform or Normal
- Inverse transformation used for MC sample. Use efficient sampling techniques in the unitary uniform uncorrelated space.

PCE model surrogate

- Polynomial chaos expansion working on the uncorrelated space.
- Trained using k-Fold validation to avoid over-fitting and prefer lower order polynomials (Least absolute shrinkage and selection operator - LASSO problem).

Logistic transformation

- Used to force fixed constrains in the outputs: i.e. to avoid overshoots.
- Can be used to smooth discontinuities and to impose only positive values.

Polynomial Chaos Expansions as wind turbine aero-elastic model surrogates

DTU 10 MW RWT

Inputs: 4D Conditionally Correlated.

Input variables
$\mathbf{x} \in \mathbb{R}^4$
$(\text{WS}; \sigma_1; \alpha; \gamma)$

→ Model $\mathcal{M}(\mathbf{x})$ →

Output variables
$\mathbf{y} = \mathcal{M}(\mathbf{x}) \in \mathbb{R}$

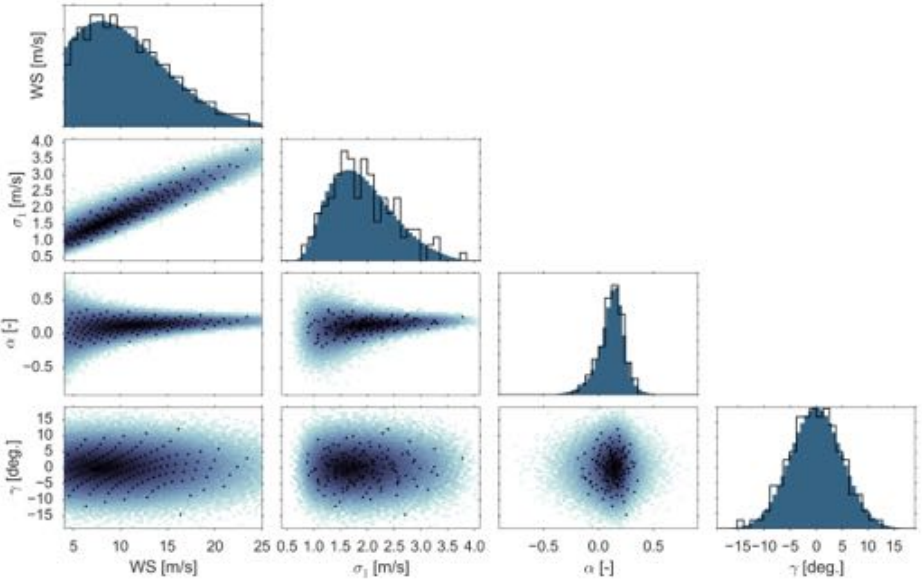
Model

- DTU: 10 MW reference WT and HAWC2 with Mann turbulence.

Uncertain Input Distribution, Class I-A site.

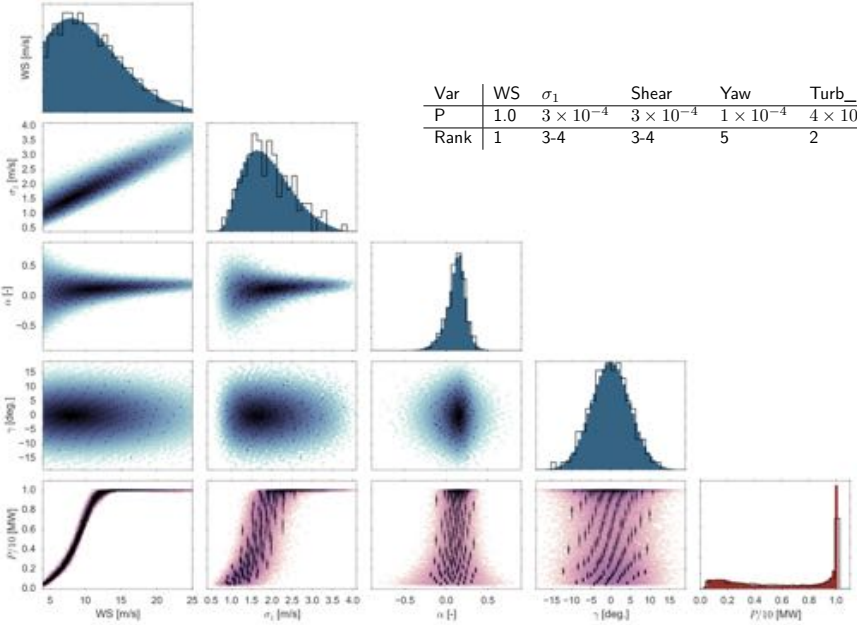
- WS $\sim Rayleigh(\mu = 10)$
- $\text{TI}_{\text{ref}} = 16\%$
- $\sigma_1 \sim Lognormal(\mu = \mu(\text{WS}, \text{TI}_{\text{ref}}), \sigma = \sigma(\text{WS}, \text{TI}_{\text{ref}}))$, NTM from IEC 61400-1.
- $\alpha \sim Normal(\mu = 0.088[\log(\text{WS}) - 1], \sigma = 1/\text{WS})$, [Dimitrov 2015]
- $\gamma \sim Normal(0, \sigma = 5°)$
- Obtain statistics from 100 TSeed realizations at each input

DTU 10 MW RWT. Training sample

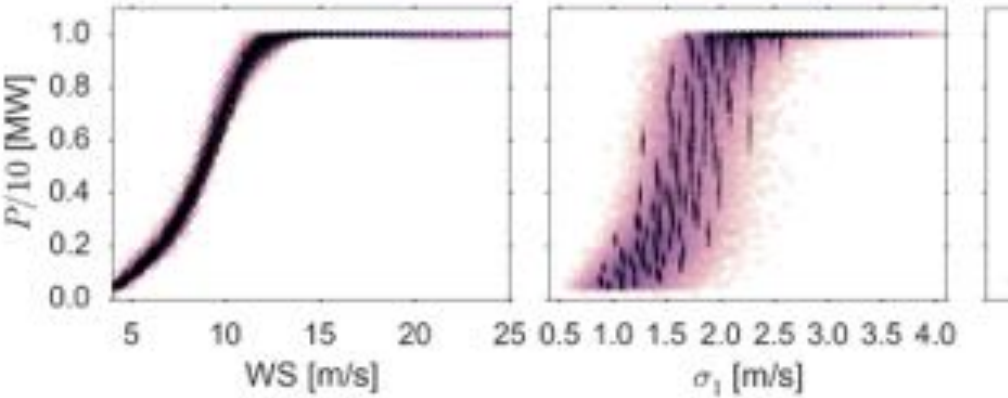


DTU 10 MW RWT. Power surrogate

Var	WS	σ_1	Shear	Yaw	Turb_real
P	1.0	3×10^{-4}	3×10^{-4}	1×10^{-4}	4×10^{-3}
Rank	1	3-4	3-4	5	2



DTU 10 MW RWT. Power surrogate



How to deal with the turbulent inflow realization uncertainty?

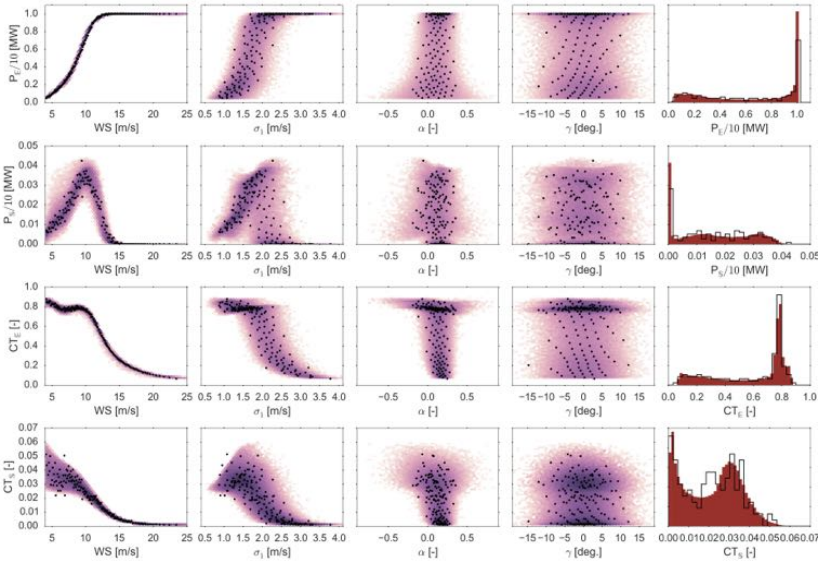
- Two polynomial chaos expansion are built independently for each output variable:

$$\hat{y}_{\mathbb{E}}(\mathbf{x}) \approx y_{\mathbb{E}}(\mathbf{x}) = \mathbb{E}(y|\mathbf{x}) \qquad \hat{y}_{\mathbb{S}}(\mathbf{x}) \approx y_{\mathbb{S}}(\mathbf{x}) = \mathbb{S}(y|\mathbf{x})$$

$$\hat{y}(\mathbf{x}) \sim \text{Normal}(\hat{y}_{\mathbb{E}}(\mathbf{x}), \hat{y}_{\mathbb{S}}(\mathbf{x}))$$

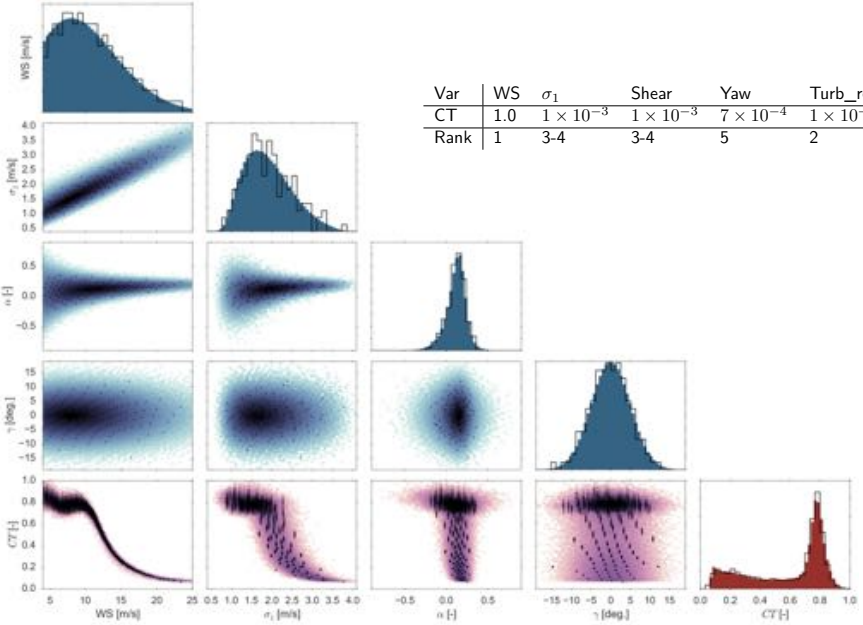
- $y_{\mathbb{S}}(\mathbf{x})$ is the local variation due to the different turbulent structures.
- For example $P_{\mathbb{S}}$ represents the standard deviation of multiple 10-min averaged powers with different turbulent inflow realization. $P_{\mathbb{S}}$ is NOT the standard deviation of the instantaneous power during the 10 minutes of simulation.

DTU 10 MW RWT. PCE for individual local statistical moments

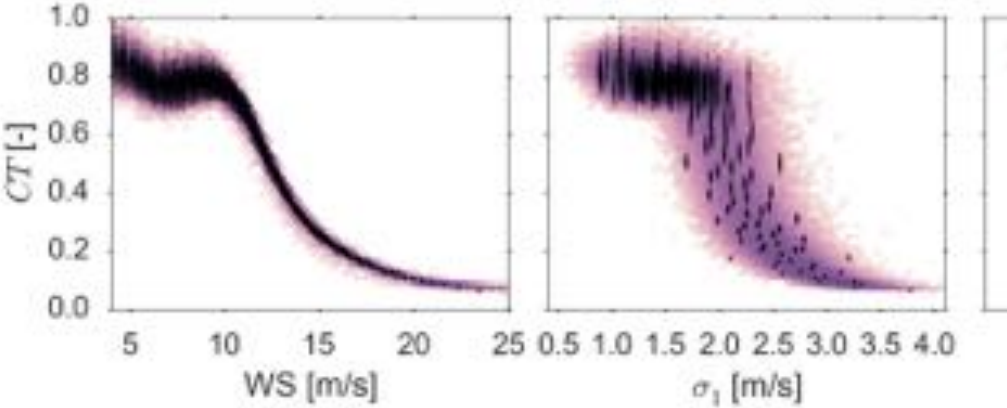


DTU 10 MW RWT. Thrust coefficient surrogate

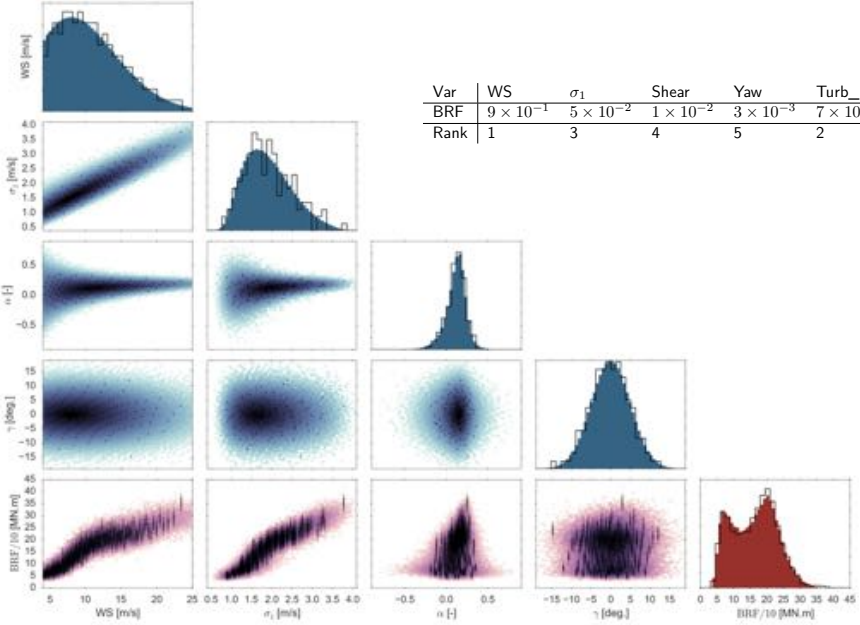
Var	WS	σ_1	Shear	Yaw	Turb_real
CT	1.0	1×10^{-3}	1×10^{-3}	7×10^{-4}	1×10^{-2}
Rank	1	3-4	3-4	5	2



DTU 10 MW RWT. Thrust coefficient surrogate

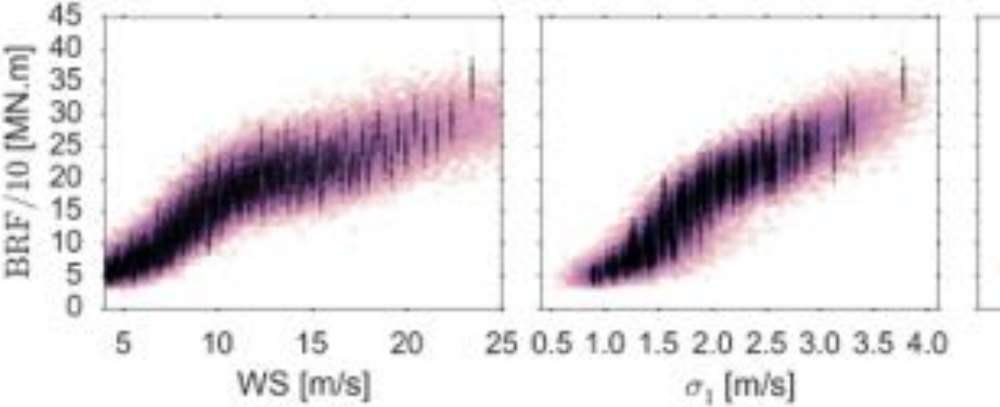


DTU 10 MW RWT. Blade root flapwise EFL surrogate

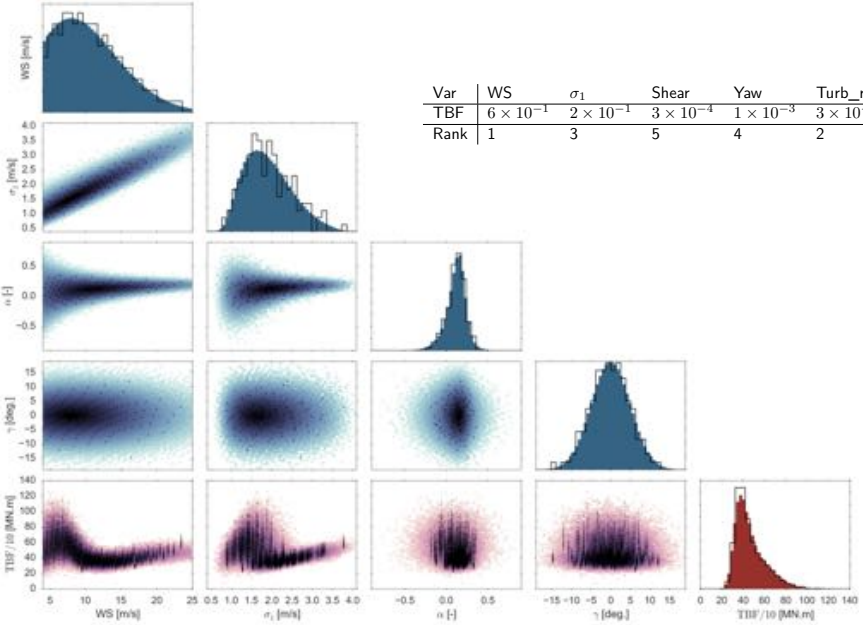


Var	WS	σ_1	Shear	Yaw	Turb_real
BRF	9×10^{-1}	5×10^{-2}	1×10^{-2}	3×10^{-3}	7×10^{-2}
Rank	1	3	4	5	2

DTU 10 MW RWT. Blade root flapwise EFL surrogate

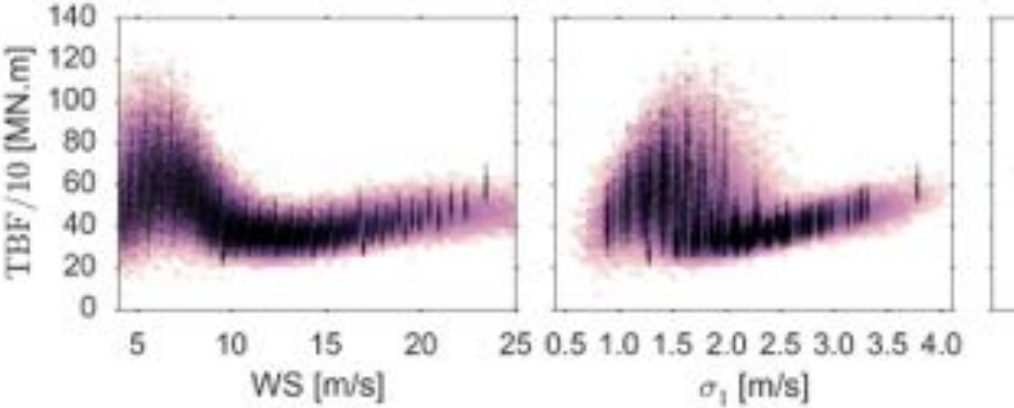


DTU 10 MW RWT. Tower bottom fore-aft EFL surrogate



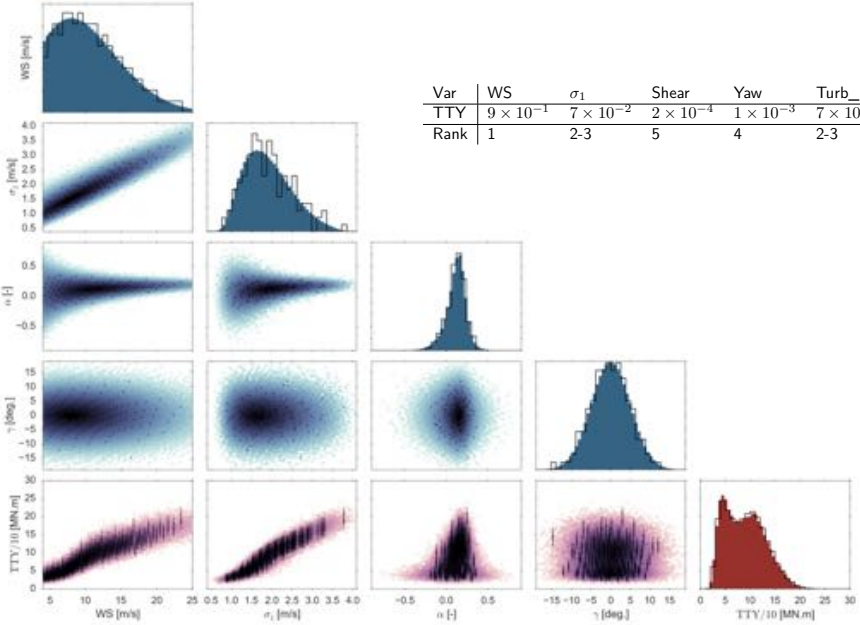
Var	WS	σ_1	Shear	Yaw	Turb_real
TBF	6×10^{-1}	2×10^{-1}	3×10^{-4}	1×10^{-3}	3×10^{-1}
Rank	1	3	5	4	2

DTU 10 MW RWT. Tower bottom fore-aft EFL surrogate

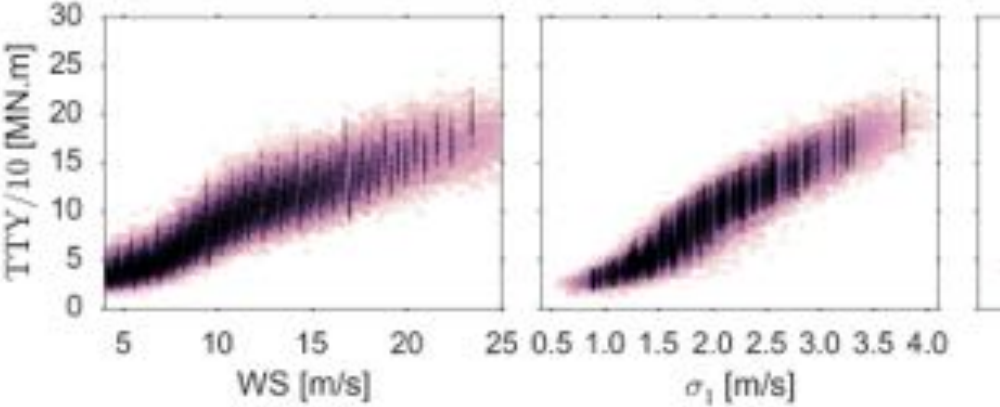


DTU 10 MW RWT. Tower top yaw EFL surrogate

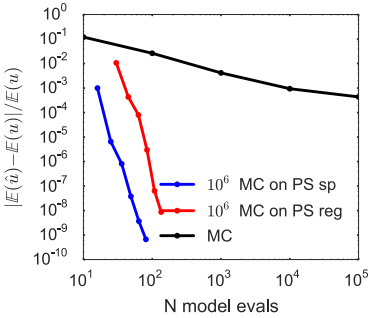
Var	WS	σ_1	Shear	Yaw	Turb_real
TTY	9×10^{-1}	7×10^{-2}	2×10^{-4}	1×10^{-3}	7×10^{-2}
Rank	1	2-3	5	4	2-3

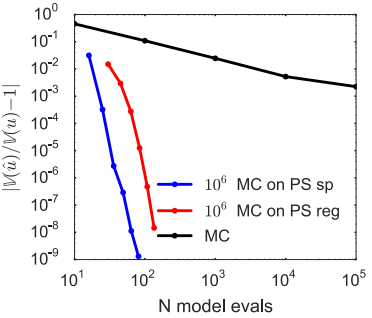


DTU 10 MW RWT. Tower bottom fore-aft EFL surrogate



Convergence example



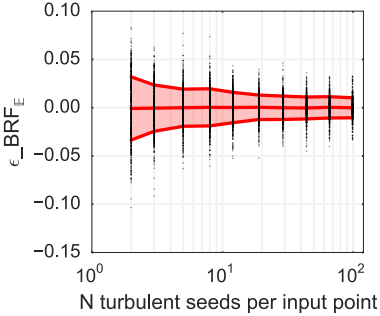


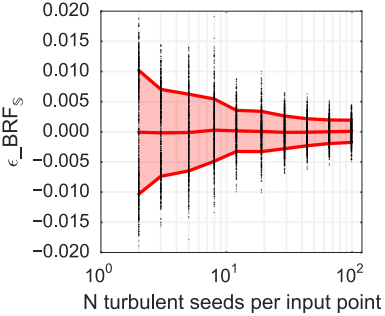
Uncertainty in the surrogates

Leave-one-out cross validation to estimate the distribution of prediction errors of the surrogates

$$\hat{y}(\mathbf{x}) \sim \text{Normal}(\hat{y}_{\mathbb{E}}(\mathbf{x}) + \epsilon_{y\,\mathbb{E}}\ \max(y),\ \hat{y}_{\mathbb{S}}(\mathbf{x}) + \epsilon_{y\,\mathbb{S}}\ \max(y))$$

$$\epsilon_{y\,\mathbb{E}} = \frac{\hat{y}_{\mathbb{E}}(\mathbf{x}_{LO}) - y_{\mathbb{E}}(\mathbf{x}_{LO})}{\max(y)} \qquad \epsilon_{y\,\mathbb{S}} = \frac{\hat{y}_{\mathbb{S}}(\mathbf{x}_{LO}) - y_{\mathbb{S}}(\mathbf{x}_{LO})}{\max(y)}$$

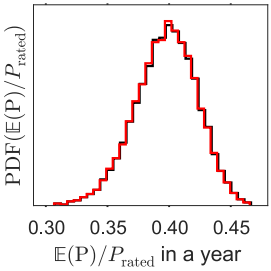


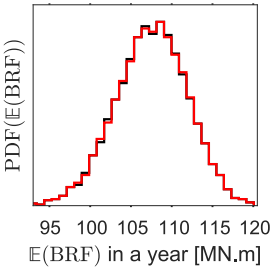


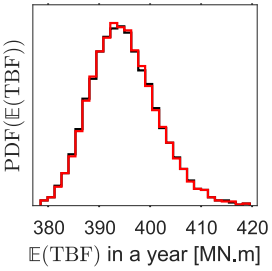
Uncertainty in AEP and E(EFL).
Two nested propagations of uncertainty.

Use the surrogate of the DTU 10 MW RWT with uncertain WS resources.
Assume a distribution of the 1 year WS resources

Variable	Distribution	Parameters	
A	Normal	$\mu_A = 9$	$\sigma = 0.5$ m/s
k	Normal	$\mu_A = 2$	$\sigma = 0.1$
$x_0 =$ WS	Weibull	scale$= A$	shape$= k$
$x_1 = \sigma_1$	Lognormal	μ_{σ_1}(WS)	σ_{σ_1}(WS)
$x_2 = \alpha$	Normal	μ_α(WS)	σ_α(WS)
$x_3 = \gamma$	Normal	$\mu_\gamma = 0$	$\sigma_\gamma = 5$ deg.







Conclusions

The surrogates are a way to obtain load estimation under site specific characteristics without sharing the proprietary aero-elastic design.

PCE as Aero-ealastic model surrogate:

- Efficient uncertainty propagation that enables to compute the statistics of the output such as: mean, standard deviation and sensitivity analysis.
- Effect of turbulent seed requires to estimate the mean and variance for every simulation using a sample of turbulent seeds for each output.
- The surrogate is able to predict both the local and global distribution of the P, CT, EFL.
- It is possible to use the surrogate inside a wind power plant optimization framework and inside uncertainty estimations of AEP and lifetime EFL.

Future Work

Surrogates

- NREL 5 MW floating RWT. Under the 5D input case [Graf 2015].
- DTU 10 MW RWT operating inside a wind farm. Dynamic wake meandering model to predict input flow conditions. 9D input case.

Sensitivity analysis and uncertainty propagation:

- PCE for SA with large number of inputs (more than 100) applied to WAsP.

Reliability and failure estimation:

- Importance sampling with PCE for extreme loads

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[Saltelli 2010] Saltelli, A., Annoni, P., Azzini, I., Campolongo, F., Ratto, M. and Tarantola, S. (2010) Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index. Computer Physics Communications, 181(2), 259–270.

Questions?

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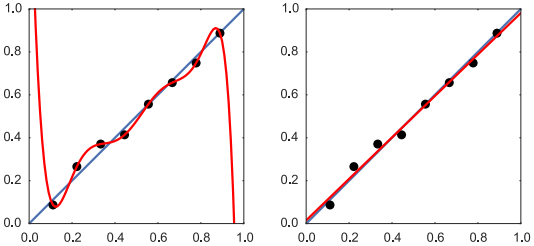
How to avoid over fitting and achieve sparsity?

Sparse linear model regression

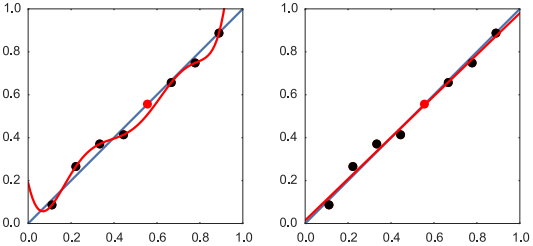
- Least Absolute Shrinkage and Selection Operator problem (LASSO) is useful to avoid over-fitting and achieve sparsity in the PCE.
- LASSO is a least squares minimization problem with a l_1 penalization on the coefficients ($\mathbf{c}$):

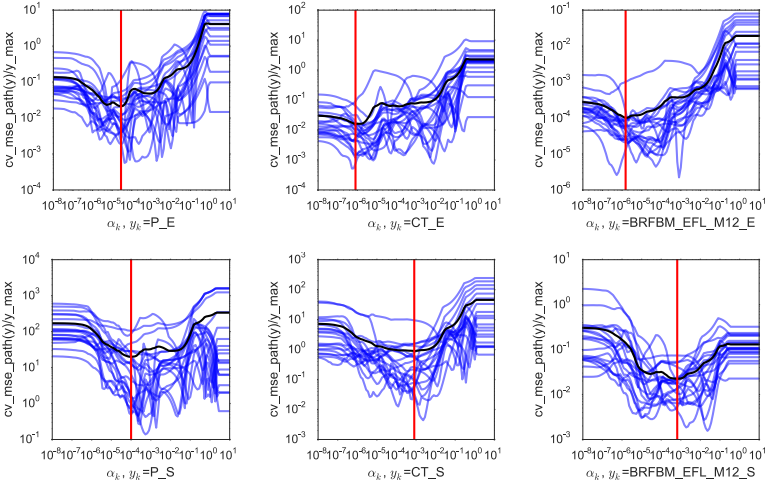
$$\min_{\mathbf{c}} \sum_{i=0}^{N-1} \left[\sum_{l=0}^{N_c-1} c_l \phi_l(\mathbf{w}_i) - y(\mathbf{x}_i) \right]^2 + \alpha \sum_{l=0}^{N_c-1} |c_l|$$



How to select the right sparsity?
k-fold cross validation

- It divides the dataset in k groups and uses k-1 groups ("folds") for training and the remaining for validation. Repeat this process until all the groups have been the validation set.
- k-fold cross validation is repeated for multiple values of the sparsity parameter (α).
- As a result it gives the optimal sparsity parameter (α).



DTU 10 MW RWT.
20-fold Cross validation examples

- Each blue line is the mean square error of a single k-fold prediction (trained with 95% of the data and tested in the remaining 5%)
- Black line is the average over all the validation set combinations