THE ANALYSIS OF EXPERIMENTAL RESULTS IN THE WINDMILL BRAKE AND VORTEX RING STATES OF AN AIRSCREW.

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(a) *Introduction.*—The vortex theory of airscrews determines the behaviour of an airscrew under ordinary working conditions but breaks down in the vortex ring state and in part of the windmill brake state. The theory may be represented in the form of a characteristic curve connecting two non-dimensional parameters F and f.

(b) *Range of Investigation.*—Experimental data have been analysed to determine the form of the characteristic curve in the regions where the vortex theory is inapplicable or inaccurate.

(c) *Conclusions.*—An empirical form of the characteristic curve has been determined which fits the experimental data and joins on to the theoretical curves in the propeller and windmill brake states. The exact form of the curve will remain somewhat uncertain until the tunnel interference is known accurately or until further experiments are available from an open jet tunnel.

1. *Introduction.*

The vortex theory of airscrews, as developed in reports R. & M. 786 and R. & M. 869, determines the behaviour of an airscrew in the ordinary propulsive state and in one part of the windmill brake state, but the theory cannot be applied to the vortex ring state or to the adjacent part of the windmill brake state. An attempt to extend the theory empirically and by means of certain general theoretical arguments has been made by Mr. Lock in report R. & M. 1014, and this discussion has revealed the general nature of the characteristic curve in the region where the vortex theory breaks down. The present report is an attempt to give precision to the characteristic curve in this region by means of the analysis of suitable experimental data.

It is convenient to express the thrust on the elements of an airscrew at radial distance r by the equations

$$\frac{dT}{dr} = 4\pi r\rho u^2 F \qquad (1)$$

$$\frac{dT}{dr} = 4\pi r\rho V^2 f \qquad (2)$$

where u is the axial velocity at the blade elements and F and f are two non-dimensional parameters. The fundamental problem

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of airscrew theory may then be conceived as the determination of the relationship connecting F and f for all working states of the airscrew. This relationship is suitable exhibited as a curve of $\frac{1}{f}$ against $\frac{1}{F}$, and this curve may be called the characteristic curve of an airscrew. F and f are essentially of the same sign and it is sufficient to plot the characteristic curve in the positive quadrant. The sign of the element of thrust is the same as that of F.

The value of F can be expressed in terms of the aerodynamic characteristics of the blade element and of the angle ϕ which the relative wind makes with the plane of rotation, in the form

$$F = \frac{bc}{4\pi r} \frac{k_L \cos\phi - k_D \sin\phi}{sin^2\,\phi} \quad \text{.................(3)}$$

where

$$\tan\phi = \frac{u}{r(\Omega - \omega)}$$

and ω is the interference angular velocity, which is usually so small that it may be neglected in calculating the value of F. The profile drag coefficient k_D is also small and for many purposes it is sufficiently accurate to calculate the value of F from the approximate formula

$$F = \frac{bc}{4\pi r} \frac{k_L}{\phi^2} \quad \text{..........................(4)}$$

By means of equations (1) and (3) the thrust is determined as a function of the axial velocity u and the final step in the calculation is to determine the forward speed V from the characteristic curve connecting F and f.

The vortex theory of airscrews determines the relationship between F and f by means of the momentum equation

$$\frac{dT}{dr} = 2\pi r\rho u(u_1 - V) \quad \text{.....................(5)}$$

where u_1 is the axial velocity in the final slipstream, and by considering the nature of the vortex system this momentum equation is transformed to

$$\frac{dT}{dr} = 4\pi r\rho u(u - V)$$

It follows that

$$f = \frac{F}{(1 - F)^2} \quad \text{..........................(6)}$$

This result refers to a propulsive airscrew and in the windmill brake state the sign of the momentum equation must be changed. The corresponding branch of the characteristic curve is

$$f = \frac{F}{(1 + F)^2} \quad \text{..............................(7)}$$

In equations (6) and (7) F is regarded as essentially positive, irrespective of the sign of the expression (3). The vortex theory of airscrews ceases to be valid when F is numerically greater than unity and is probably of poor accuracy when F is slightly less than this limiting value.

The conclusions drawn from the vortex theory of airscrews are based on the assumption that the interference velocity at the airscrew disc may be calculated on the assumption that the vortex system of the slipstream extends backwards from the airscrew without any contraction. More recently an attempt has been made to estimate the effect of the contraction of the slipstream* and the characteristic curve in the propulsive state of an airscrew is found to agree sensibly with the formula proposed by Mr. Lock.

$$\frac{1}{f} = \frac{1}{F} - 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

This modified theory agrees closely with the vortex theory for small thrust but departs from it slightly as the thrust increases.

The characteristic curves determined by the vortex theory and by the modified theory are shown in Fig. 1. The object of the subsequent analysis is to determine an empirical curve covering the range in which the theoretical curves are invalid or inaccurate. Suitable experimental data for this purpose are contained in reports R. & M. 885 and R. & M. 1014 and in N.A.C.A. Note 221 which record the thrust of an airscrew whose blades are formed of aerofoils of constant chord, section, and angle.

2. *Method of Analysis.*

Consider an airscrew formed of b identical blades of constant chord c and constant blade angle β. If the blades extend from radial distance ϵR to R, the total blade area is

$$S = bcR\,(1 - \epsilon)$$

and the solidity is

$$\sigma = \frac{S}{\pi R^2} = \frac{bc(1 - \epsilon)}{\pi R}$$

If a_0 is the angle of no lift of the aerofoil section forming the blades, the blade angle measured from the no lift line is

$$\theta = a_0 + \beta$$

and when the airscrew is working at a negative rate of advance with an axial velocity u through the disc the angle of incidence of the blade element at radial distance r is

$$a = \theta + \phi = \theta + \frac{u}{r\Omega}$$

and the corresponding lift coefficient will be taken to be

$$k_L = 3\,(\theta + \phi).$$

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* This analysis will form the subject of a separate report.

Assuming the axial velocity u to be uniform over all the blade elements, and writing

$$u = xR\Omega$$

the thrust of the airscrew is obtained as

$$\begin{aligned} T &= \int_{\epsilon R}^{R} 3bc\rho r^2\Omega^2 (\theta + \phi)\, dr \\ &= \int_{\epsilon R}^{R} 3bc\rho\Omega^2 (\theta r^2 + xrR)\, dr \\ &= bc\rho\Omega^2 R^3 \left\{ \theta(1 - \epsilon^3) + \frac{3}{2}x(1 - \epsilon^2) \right\} \end{aligned}$$

$$\frac{T}{\pi R^2 \rho \Omega^2 R^2} = \frac{\sigma}{1 - \epsilon} \left\{ (\alpha_0 + \beta)(1 - \epsilon^3) + \frac{3}{2} x (1 - \epsilon^2) \right\} \quad \ldots(9)$$

with the alternative expressions

$$\left.\begin{aligned} k_T &= \frac{T}{\rho n^2 D^4} = \frac{\pi^3}{4} \frac{T}{\pi R^2 \rho \Omega^2 R^2} \\ \frac{T}{\pi R^2 \rho V^2} &= \left(\frac{\Omega R}{V}\right)^2 \frac{T}{\pi R^2 \rho \Omega^2 R^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(10)$$

Equation (9) determines the value of the parameter x in terms of the thrust T and the rate of rotation Ω since all the other quantities involved in the equation are known.

But by integration of equations (1) and (2) we have

$$\begin{aligned} T &= 2\pi R^2(1 - \epsilon^2)\rho u^2 F \\ &= 2\pi R^2(1 - \epsilon^2)\rho V^2 f \end{aligned}$$

and hence

$$\left.\begin{aligned} f &= \frac{1}{2(1 - \epsilon^2)} \frac{T}{\pi R^2 \rho V^2} \\ F &= \frac{1}{2(1 - \epsilon^2)x^2} \frac{T}{\pi R^2 \rho \Omega^2 R^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(11)$$

Thus F is determined as a function of T and Ω, and f is determined as a function of T and V.

3. *Analysis of N.A.C.A. Note* 221.

This note records the drag of a number of windmills rotating freely at zero torque. The principal characteristics were

$$b = 2 \text{ and } 4 \quad c = 7{\cdot}7 \quad R = 30 \quad \epsilon = \tfrac{1}{2}$$

so that $\sigma = 0{\cdot}0408\, b$.

The aerofoil section was Durand 13, whose no lift angle is 6° or $\alpha_0 = 0{\cdot}105$, and the blade angle β was varied over the range 1° to − 15°.

From the tabulated results we derive at once the values of

$$\lambda = \frac{V}{\Omega R} \text{ and } k_D = \frac{T}{\pi R^2 \rho V^2}$$

and equations (9) to (11) then lead to the following formulæ for the analysis

$$\left.\begin{aligned} x &= 10\cdot 9\,\frac{\lambda^2 k_D}{b} - \frac{7}{9}\beta - 0\cdot 082 \\ \frac{1}{f} &= \frac{1\cdot 5}{k_D} \\ \frac{1}{F} &= \left(\frac{x}{\lambda}\right)^2 \frac{1}{f} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots(12)$$

The full details of this analysis are given in Table 1. The values of f and F are also shown in Fig. 1, but for small values of β the points represent the mean of a group of three angles instead of the individual results in order to even out accidental variations.

These experimental results were obtained in an open jet wind tunnel for which the tunnel interference on an airscrew is believed to be negligible. The results therefore represent free air conditions.

4. *Analysis of Report R. & M.* 885.

This report contains the drag of four windmills rotating freely at zero torque. The principal characteristics were

$$b = 2 \quad c = 2\cdot 5 \quad R = 18 \quad \epsilon = \frac{1}{6}$$

so that $\sigma = 0\cdot 0737$.

The aerodynamic data of the aerofoil section are included in the report, and by taking the slope of the lift curve above $k_L = 0\cdot 14$ the no lift angle is found to be $2^\circ\cdot 6$ or $\alpha_0 = 0\cdot 045$.

As in the previous case, the experiments determine the value of λ and k_D, and the formulæ for the analysis become

$$\left.\begin{aligned} x &= 7\cdot 75\,\lambda^2 k_D - 0\cdot 683\,\beta - 0\cdot 031 \\ \frac{1}{f} &= \frac{35}{18}\frac{1}{k_D} \\ \frac{1}{F} &= \left(\frac{x}{\lambda}\right)^2 \frac{1}{f} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots(13)$$

Mr. Lock has provided further details of the original measurements which have made it possible to analyse the experiments in two different ways, using

(1) the uncorrected velocity as given by the ordinary tunnel gauge;

(2) the corrected velocity obtained from the plane of the airscrew disc.

The details of the analysis are given in Table 2. The correction to the velocity does not alter the values of x or F, but it increases the value of $\frac{1}{f}$.

The corrected values are plotted in Fig. 1 and agree closely with the American results, showing therefore that the method of estimating the equivalent free air speed for experiments in a closed tunnel from the speed in the plane of the airscrew disc is reasonably accurate.

5. *Analysis of Report R. & M.* 1014.

The experiments described in this report were obtained with the airscrew described in Report R. & M. 885, but different blade angles were used and the experiments covered a wide range of rates of advance of the airscrew. The results are given as the values of k_T and J based on the uncorrected tunnel speed, and the appropriate formulæ for the anlaysis are

$$\left.\begin{aligned} x &= k_T - 0{\cdot}683\,\beta - 0{\cdot}031 \\ \frac{1}{f} &= 1{\cdot}52\,\frac{J^2}{k_T} \\ \frac{1}{F} &= 15{\cdot}0\,\frac{x^2}{k_T} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(14)$$

All the experimental points have been analysed by this method and the results are shown in Fig. 2, together with the uncorrected values from Report R. & M. 885. The results determine a mean curve with fair accuracy, although they are rather scattered in the windmill brake state. The mean curve in the windmill brake state is possibly a little lower than the curve given by the experiments of Report R. & M. 885.

6. *The characteristic curve.*

In order to deduce the best empirical curve connecting F and f, it is necessary to know the magnitude of the wind tunnel interference on the experimental results shown in Fig. 2. Although the exact magnitude of this correction is still rather uncertain, we know that its effect is to raise the experimental points in the windmill brake state and to lower the points in the propeller state.

In the windmill brake state the experimental points deduced from R. & M. 885 are brought into agreement with the American results from an open jet channel by using the speed in the plane of the airscrew disc instead of the uncorrected tunnel speed. This method of correction may therefore be accepted as reasonably accurate in this range and the empirical curve deduced from the observations is then of the form shown in Fig. 1. It touches the ordinate axis at $\frac{1}{f} = 2$ and runs smoothly into the theoretical curve of the vortex theory at $\frac{1}{F} = 2$. This curve is also shown in Fig. 2 in order to indicate the magnitude of the tunnel interference.

In the propeller state the experimental points shown in Fig. 2 $\left(\frac{1}{F} > 2\right)$ will be lowered when the tunnel correction is applied. The experiments suggest that the static condition is represented by $\frac{1}{F} = 2$, which is in accordance with the approximate formula suggested by Mr. Lock and with the indications of the modified theory obtained by considering the effect of the contraction of the slipstream. This value has been accepted in drawing the characteristic curve, and in the propeller state the characteristic curve has been taken to be that given by equation (8).

Finally in the vortex ring state the magnitude, and even the sign, of the tunnel correction are quite unknown but the experimental points suggest that the propeller and windmill brake branches of the characteristic curve may be joined suitably by the curve shown in Figs. 1 and 2.

The characteristic curve in free air now has the form shown in Figs. 1 and 2, which fits the experimental points with good accuracy, when the tunnel interference correction is applied, over the whole range.

The characteristic curve has been deduced from experiments with airscrews whose blades are formed of aerofoils of constant chord and incidence. It may be checked by using the curve to estimate the characteristics of airscrews of conventional design, but cannot be established finally until the tunnel interference correction is known accurately for all working states or until sufficient experiments are available from an open jet type of tunnel.

The empirical part of the characteristic curve is defined by the following co-ordinates :—

$\frac{1}{F} =$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	2	3	4
$\frac{1}{f}$	2·00	1·08	0·80	0·60	0·50	0	0·64	1·73
	2·00	2·87	3·17	3·41	3·63	4·50	5·33	6·25

TABLE 1.

Analysis of N.A.C.A. Note 221.

b	β	λ	k_D	x	$\frac{1}{f}$	$\frac{1}{F}$
4	1°	0·266	0·630	0·026	2·38	0·023
	0	0·241	0·638	0·019	2·35	0·015
	− 1	0·245	0·625	0·034	2·40	0·046
	− 2	0·230	0·632	0·036	2·37	0·058
	− 3	0·217	0·614	0·038	2·45	0·075
	− 4	0·204	0·595	0·040	2·52	0·097
	− 5	0·194	0·593	0·047	2·53	0·148
	−10	0·176	0·490	0·095	3·06	0·895
2	0	0·183	0·552	0·019	2·72	0·029
	− 1	0·170	0·566	0·021	2·65	0·040
	− 2	0·157	0·589	0·024	2·55	0·060
	− 3	0·157	0·547	0·033	2·74	0·121
	− 4	0·152	0·538	0·040	2·79	0·194
	− 5	0·150	0·506	0·048	2·96	0·303
	−10	0·139	0·360	0·092	4·17	1·83

TABLE 2.

Analysis of Report R. & M. 885.

(1) Uncorrected velocity (tunnel gauge).

β	λ	k_D	x	$\frac{1}{f}$	$\frac{1}{F}$
− 6°	0·092	0·408	0·067	4·78	2·55
− 3	0·0855	0·634	0·041	3·07	0·70
− 1	0·090	0·774	0·030	2·52	0·28
− ½	0·0905	0·820	0·027	2·37	0·21

(2) Corrected velocity (plane of airscrew disc).

β	λ	k_D	x	$\frac{1}{f}$	$\frac{1}{F}$
− 6°	0·093	0·396	0·067	4·91	2·55
− 3	0·088	0·598	0·041	3·25	0·70
− 1	0·098	0·660	0·030	2·95	0·28
− ½	0·0995	0·381	0·027	2·86	0·21

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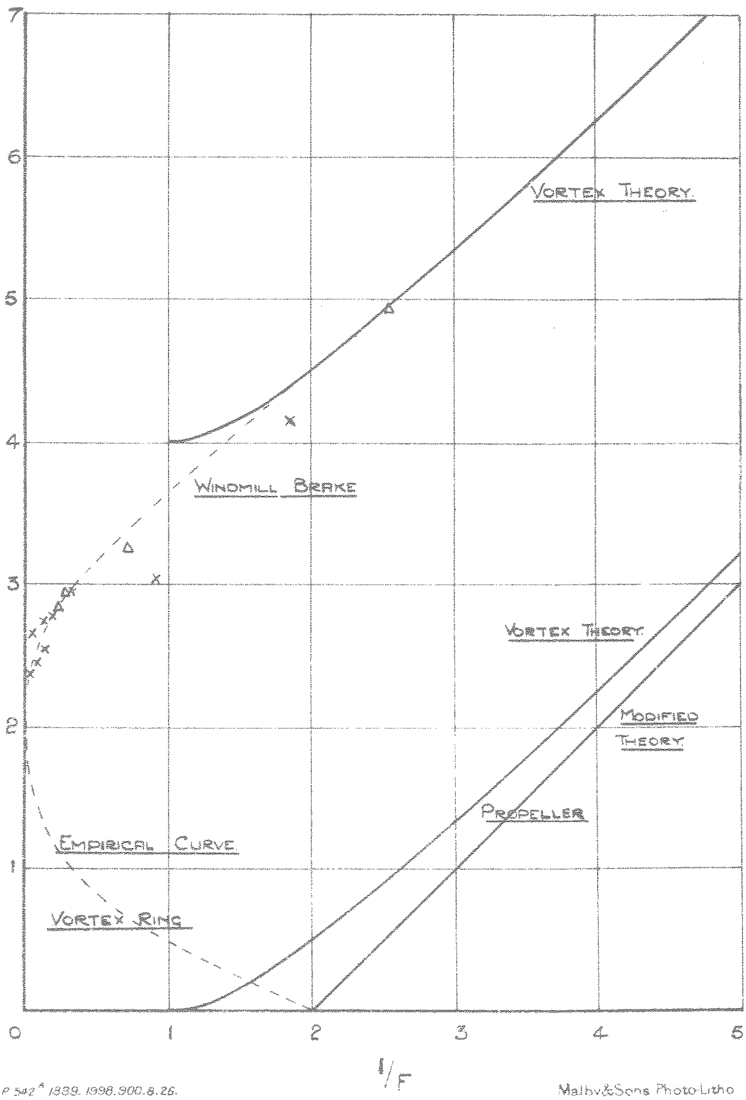
Analysis of Windmill Experiments

Results Based on Corrected Tunnel Speed.

$$\frac{dT}{dr} = 4\pi r \rho u^2 F = 4\pi r \rho V^2 f$$

× N.A.C.A. Note 221.

△ R & M. 885 (Corrected)



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ANALYSIS OF WINDMILL EXPERIMENTS

RESULTS BASED ON UNCORRECTED TUNNEL SPEED.

$$\frac{dT}{dr} = 4\pi r \rho u^2 F = 4\pi r \rho V^2 f.$$